Cosmological perturbation theory to second order for curvature, density and gravity waves on FRW background; and the WMAP results of inhomogeneity and clustering in the early universe.

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Abstract

The second order perturbation calculations for gravity wave and Einstein equation for space time and matter are presented for the FRW metric cosmological model. While exact equations are found, suitable approximations are made to obtain definite results. In the gravity wave case the small wavelength case allows nearly locally flat background for obtaining a fit to the WMAP data. In the density and curvature case the FRW background is retained for the length scale of WMAP. Clustering and inhomogeneity are understood. The gravity wave ripples from Big Bang couple nonlinearly and redistribute the modes to higher values of 'l' giving consistency with the WMAP results. The order by order consistency of Einstein equations relate the second order perturbations in the curvature and density and the wrinkles in spacetime caused by the gravity wave modes reorganize these distributions. The radiation data of WMAP gives the picture of a FRW spacetime deformed and wrinkled consistent with matter distribution to one hundred thousandths parts variation.

1 INTRODUCTION

The WMAP data at the observable horizon of 300000 years after Big Bang gives confirmation of anisotropy and inhomogeneity in the universe. A bubble model during inflation is made with composite equation of state. An explanation of the anisotropy with WMAP $C(l) vs" l"$ is made with gravity wave mixing causing 'wrinkled' space time from big bang to WMAP epoch. A density perturbation model within a Newtonian and Einsteinian analysis is presented. The clustering model attempts to explain the two fluid phase with cluster formation at different length scales. The recent WMAP picture shows that $\frac{\triangle\rho}{\rho}and\frac{\triangle T}{T}$ of the order of 10^{-5}.Then it evolves to clustering and clumping of structures.There are number of models for this clustering.In this project as a continuation of Part A and Part B.The density perturbations are evolved till 1 billion years from Big Bang when galaxies are formed.Therefore we simulate the process of gravitational clustering as process of diffusion running backwards in time i.e, with an anti-diffusion model.A simple model of a uniform Gaussian distribution which can collapse to localized sharp distributions simulates the process of variation in density.We have constructed a 1-d Gaussian and a 2-d Gaussian depicting variation of density on a line and in a plane respectively.Based on this a 3-d Gaussian representing

variation of density in volume may be understood.We are actually using Gaussian distributions as solutions of a diffusion equation with the following parameters the width of the Gaussian,the center of Gaussian and height of Gaussian.With these parameters a well spread Gaussian becomes narrower with time running backward and becomes wider with time running forward.Therefore we simulate the process of gravitational clustering as process of diffusion running backwards in time i.e, with an anti-diffusion model.This simulation is initially applied at the scale (4000 mega parsec) of the Universe leading to formation osuperclusters.Then within a supercluster we understand the process similarly to explain the formation of clusters.Finally inside a cluster we associate the formation of galaxies with a similar argument.

The recent WMAP picture shows us that during its initial phase there was no differentiation between various forms of matter namely dark matter,dark energy,normal matter and radiation.But as the Universe evolved different forms of matter started differentiating among themselves and occupied different locations.To be more specific the redistribution of normal matter was more pronounced as large lumps of matter started accumulating together to form centers of galaxies which are surrounded by dark matter halos. Each halo is five times more massive than the normal matter at the center.Note that the normal matter at the center together with the dark matter halo forms the respective galaxy .Some of these galaxies came together to form clusters of galaxies and many such clusters approached each other to form super cluster of galaxies.The dark energy as well as dark matter occupied in the empty space which are called voids.The redistribution of dark energy is not as much as that of normal matter.Therefore an early Universe which had a uniform distribution of various forms of matter ended up with a one in which the various forms of matter got segregated at different places.The title of our report precisely calls for an answer to this observation.

The overall properties of the Universe roughly speaking are close to being homogenous and yet telescopes reveal a wealth of detail on scales varying from single galaxies to large scale structures of size exceeding 100 mega parsecs.The existence of these cosmological structures tells us something important about the initial conditions of the Big Bang,and about physical processes that have operated subsequently.If the initial Big Bang would have been homogenous then the presently observed distribution of galaxies cannot be accounted for considering the age of the Universe.If the elementary particles in the Universe started out with a uniform distribution,then purely statistical fluctuations would occur on all scales,from which matter would eventually condense.However the highly improbable fluctuations of galactic dimensions is extremely slow and takes far more time compared to the age of the Universe.Therefore studying models with inhomogeneities is taken up.The aim of studying inhomogeneities is to understand the processes that caused the Universe to depart from uniform density The second order perturbation calculations for gravity wave and Einstein equation for space time and matter are presented for the FRW metric cosmological model. While exact equations are found, suitable approximations are made to obtain definite results. In the gravity wave case the small wavelength case allows nearly locally flat background for obtaining a fit to the WMAP data. In the density and curvature case the FRW background is retained for the length scale of WMAP. Clustering and inhomogeneity are understood.

2 Bubble model

Introduction

A model of the expansion of the Universe by the expansion of a bubble in some surrounding medium when the contents of the bubble are heated is made. The equations of evolution of the bubble radius thus obtained will be considered as the equations for evolution of the 'scale factor' of the Universe.

It is hoped that the dynamics obtained helps to shed light on modeling of inflation, dark matter/energy etc.

We also attempt to find the equation of state of the matter in the Universe if it is assumed to evolve according to the dynamics obtained. These equations of state are analysed to find hints of dark matter, dark energy and the like.

2.1 General heating of the bubble

Consider a spherical bubble containing an ideal gas in a surrounding medium of some liquid - e.g. water - at a

pressure and temperature P_0 and T_0 . Let the surface tension at the interface be S and for the gas in the bubble let $\frac{C_P}{R_N} = \alpha$. At an instant of time say $t = 0$ heat from a power source – like a laser beam or a spark plug – is incident inside the bubble causing it to expand. Let us consider the dynamics of such a bubble-

Now for any radius R of the bubble we have for the gas inside.

$$P = P_0 + \frac{2S}{R} \;\; \Rightarrow \;\; dP = -\frac{2S}{R^2}dR \tag{1}$$

From the conservation of energy for the bubble we have–

$$dQ = nC_v dT + SdA + PdV \tag{2}$$

We also have for the bubble–

$$\begin{aligned} A = 4\pi R^2 \;\; &\Rightarrow \;\; dA = 8\pi R dR \\ V = \frac{4}{3}\pi R^3 \;\; &\Rightarrow \;\; dV = 4\pi R^2 dR \end{aligned} \tag{3}$$

For the ideal gas in the bubble we have–

$$T = \frac{PV}{nR_N} \;\Rightarrow\; dT = \frac{PdV + VdP}{nR_N} \tag{4}$$

Using eqn (4) in (2) –

$$\begin{aligned} dQ &= \frac{C_V}{R_N}(PdV + VdP) + SdA + PdV \\ &= (PdV)\alpha + (VdP)(\alpha - 1) + SdA \end{aligned} \tag{5}$$

Now we substitute eqns (1) and (3) in (5)

$$dQ = \alpha(P_0 + \frac{2S}{R})(4\pi R^2 dR) + (\alpha - 1)(\frac{4}{3}\pi R^3)(-\frac{2S}{R^2}dR) + S(8\pi R dR)$$

Dividing throughout by dt and using $W(t) = \frac{dQ}{dt}$ –

$$W(t) = \dot{R}\left(\alpha(P_0 + \frac{2S}{R})(4\pi R^2) + (\alpha - 1)(\frac{4}{3}\pi R^3)(-\frac{2S}{R^2}) + S(8\pi R)\right)$$

i.e –

$$\dot{R} = \frac{W(t)}{4\pi R\left(\alpha P_0 R + \frac{4S}{3}(\alpha + 2)\right)} \tag{6}$$

2.2 Case1 - Constant power source

We consider a special case of eqn (6) when the power supplied is a constant. So we have –

$$W(t) = W_0 \tag{7}$$

Then we see that we get the following equation which can be readily integrated–

$$\int 4\pi R \left(\alpha P_0 R + \frac{4S}{3}(\alpha + 2) \right) dR = W_0 \int dt$$

to give–

$$\frac{R^2}{3} \left(\alpha P_0 R + 2S(\alpha + 2) \right) = \frac{W_0}{4\pi} t + \kappa \tag{8}$$

2.3 Case2 - Bubble conducting heat

Now lets consider a special case, where the bubble conducts heat across its surface to the surrounding reservoir. Assume that the reservoir is large enough so that it is unaffected by the heat absorbed or released by the bubble.

Let the rate of heat conduction be proportional to the area and the temperature difference across the bubble surface. So we have –

$$W = W_C = -\beta R^2(T - T_0) \tag{9}$$

We substitute for the T's using the eqns (1) (3) and (4) –

$$\begin{aligned} T &= \frac{1}{nR_N}(P_0 + \frac{2S}{R})(\frac{4}{3}\pi R^3) \\ &= \frac{4\pi}{3nR_N}(P_0R^3 + 2SR^2) \end{aligned} \tag{10}$$

Using eqns (9) and (10) we get–

$$W_C = -\frac{4\pi}{3nR_N}\beta R^2 \left(P_0R^3 + 2SR^2 - P_0R_0^3 - 2SR_0^2\right)$$

which reduces to –

$$W_C = -\frac{4\pi}{3nR_N}\beta R^2 \left(P_0(R^3 - R_0^3) + 2S(R^2 - R_0^2)\right)$$

$\Rightarrow$

$$W_C = -\frac{4\pi}{3nR_N}\beta R^2(R - R_0) \left(P_0R^2 + (P_0R_0 + 2S)(R + R_0)\right) \tag{11}$$

Using (11) in (6)

$$\dot{R} = -\frac{\beta}{3nR_N}R(R - R_0)\frac{P_0R^2 + (P_0R_0 + 2S)(R + R_0)}{(\alpha P_0)R + \frac{4S}{3}(\alpha + 2)} \tag{12}$$

We note that the sudden increase in $R(t)$ is *very similar to the inflation scenario in conventional Cosmology.*

2.4 Other models

We shall consider variants of previously developed models and compare them to existing cosmological models.

2.4.1 Hybrid model with inflation

We consider a model where the bubble first expands rapidly according to eqn (12) but then eqn (8) takes over. This involves a discontinuous change in the function $W(t)$ from that given by eqn (9) to eqn (7). This can be considered to be *analogous to a phase transition that is presumed to have happened during inflation*.

2.4.2 Single model with inflation

We shall use a variant of eqn (12) where R_0 is no longer considered a constant but a function of time. We note that in such a case $R(t)$ initially undergoes a sudden *inflation* and then follows exactly the functional form given by $R_0(t)$.

Thus we use the Friedmann evolution as the form for $R_0(t)$ and so we have an *initial inflation followed by a Friedmann evolution!!!*

3 Gravitational Waves

Introduction

Here we shall attempt to solve the equations of General Relativity for the case of small perturbations in the background metric and obtain wave solutions.

Calculations are done upto 1^{st} and 2^{nd} order in the perturbations.

We hypothesize that *a primordial gravitational wave is responsible for the anisotropy in the CMBR as observed by WMAP*. Starting from a isotropic primordial gravitational wave, we hope to make use of the non-linear coupling in a gravitational wave solution — *upto* 2^{nd} *order* — to evolve the wave into the anisotropies seen in the WMAP data.

3.1 Equations of General Relativity

We list here the equations of General Relativity that shall be useful in this calculation.

Let us consider a metric g_{ik} on spacetime; then we have the following –

- The Christoffel symbols are

$$\Gamma^i{}_{kl} = \frac{1}{2} g^{im} (g_{mk,l} + g_{ml,k} - g_{lk,m}) \tag{13}$$

- The Riemann tensor is

$$R^i{}_{klm} = \Gamma^i{}_{km,l} - \Gamma^i{}_{kl,m} + \Gamma^i{}_{nl} \Gamma^n{}_{km} - \Gamma^i{}_{nm} \Gamma^n{}_{kl} \tag{14}$$

- The Riemann tensor has following symmetries

$$R_{iklm} = -R_{kilm} = -R_{ikml} = R_{lmik}$$

$$R_{iklm} + R_{ilmk} + R_{imkl} = 0 \tag{15}$$

$$R^n{}_{ikl;m} + R^n{}_{ilm;k} + R^n{}_{imk;l} = 0$$

- The Ricci tensor is given by

$$R_{ik} = R^l{}_{ilk} \tag{16}$$

- The scalar curvature is

$$R = R^i{}_i = g^{ik} R_{ik} \tag{17}$$

- Given the energy-momentum tensor T_{ik} Einstein equation can be written as

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi k}{c^4} T_{ik}$$

$$R_i{}^k - \frac{1}{2} \delta_i{}^k R = \frac{8\pi k}{c^4} T_i{}^k \tag{18}$$

$$R_{ik} = \frac{8\pi k}{c^4} (T_{ik} - \frac{1}{2} g_{ik} T)$$

3.2 Perturbations

To analyze the phenomena of gravitational waves, a perturbation h_{ik} in the metric of spacetime g_{ik} over its "background" value $g_{ik}^{(0)}$ is considered as –

$$g_{ik} = g_{ik}{}^{(0)} + h_{ik} \tag{19}$$

It is chosen by convention to "raise" and "lower" indices using the "background" metric $g_{ik}^{(0)}$. That is - $h_k^m = g^{mi(0)} h_{ik}$, where –

$$g^{mi}{}^{(0)} g_{ik}{}^{(0)} = \delta_k^m \tag{20}$$

The perturbations in the contravariant metric is evaluated by solving the equation –

$$g^{mi} g_{ik} = \delta_k^m \tag{21}$$

Let – $g^{mi} = g^{mi}{}^{(0)} + f^{mi}$. Then substitution into above equation gives –

$$\left(g^{mi}{}^{(0)} + f^{mi}\right)\left(g_{ik}{}^{(0)} + h_{ik}\right) = \delta_k^m$$

i.e.

$$h_k^m + f_k^m + f^{mi} h_{ik} = 0 \tag{22}$$

Now, upto first order in h we have –

$$f^{mk}{}^{(1)} = -h^{mk}$$

Substituting this back in Eq.(40) we find the next order as –

$$f^{mk}{}^{(2)} = h^{mi} h_i^k$$

Similarly we can iteratively find the higher orders of f as –

$$f^{mk}{}^{(n)} = -f^{mi}{}^{(n-1)} h_i^k \quad ; \quad f^{mk}{}^{(1)} = -h^{mk} \tag{23}$$

3.3 Ricci Tensor

The Riemann tensor is given as –

$$R^{i}{}_{klm} = \frac{1}{2} g^{in} \left(g_{nm,kl} + g_{kl,nm} - g_{nl,km} - g_{km,nl} \right) \tag{24}$$

The Ricci tensor is thus –

$$R_{km} = R^{l}{}_{klm} = \frac{1}{2} g^{ln} \left(g_{nm,kl} + g_{kl,nm} - g_{nl,km} - g_{km,nl} \right) \tag{25}$$

The Ricci tensor is now computed upto 2^{nd} order in the perturbation h.

$$\frac{1}{2} \left({g^{ln}}^{(0)} - h^{ln} + h^{lp} h^{n}_{p} \right) \left(\begin{array}{c} {g_{nm,kl}}^{(0)} + {g_{kl,nm}}^{(0)} - {g_{nl,km}}^{(0)} - {g_{km,nl}}^{(0)} \\ + h_{nm,kl} + h_{kl,nm} - h_{nl,km} - h_{km,nl} \end{array} \right)$$

Now separating the Ricci tensor into parts of different orders we have –

$${R_{km}}^{(0)} = \frac{1}{2} {g^{ln}}^{(0)} \left({g_{nm,kl}}^{(0)} + {g_{kl,nm}}^{(0)} - {g_{nl,km}}^{(0)} - {g_{km,nl}}^{(0)} \right) \tag{26}$$

$$\begin{aligned} {R_{km}}^{(1)} = & \frac{1}{2} {g^{ln}}^{(0)} \left(h_{nm,kl} + h_{kl,nm} - h_{nl,km} - h_{km,nl} \right) \\ & - \frac{1}{2} h^{ln} \left({g_{nm,kl}}^{(0)} + {g_{kl,nm}}^{(0)} - {g_{nl,km}}^{(0)} - {g_{km,nl}}^{(0)} \right) \end{aligned} \tag{27}$$

$$\begin{aligned} {R_{km}}^{(2)} = & -\frac{1}{2} h^{ln} \left(h_{nm,kl} + h_{kl,nm} - h_{nl,km} - h_{km,nl} \right) \\ & + \frac{1}{2} h^{lp} h^{n}_{p} \left({g_{nm,kl}}^{(0)} + {g_{kl,nm}}^{(0)} - {g_{nl,km}}^{(0)} - {g_{km,nl}}^{(0)} \right) \end{aligned} \tag{28}$$

3.4 1^{st} order calculation with a Minkowskian background

Now we shall now proceed with the calculations with following assumptions –

1. The unperturbed background spacetime is Minkowskian.

   $\Rightarrow\ {g_{ik}}^{(0)} = \eta_{ik} \equiv$ Minkowskian metric

2. Only terms upto 1^{st} order in h_{ik} or its derivatives are significant. All higher order terms are negligible.

For a Minkowskian background we also have ${R^i{}_{klm}}^{(0)} = 0$; so we need to consider only the 1^{st} order term of the Riemann tensor
. From Eq. (27), the Ricci tensor of 1^{st} order is –

$$ {R_{km}}^{(1)} = \frac{1}{2}\eta^{ln}\left(h_{nm,kl} + h_{kl,nm} - h_{nl,km} - h_{km,nl}\right) \tag{29} $$

which reduces to –

$$ R_{km} = \frac{1}{2}\Box h_{km} + \frac{1}{2}(h^l_{m,k,l} - h_{,k,m} + {h_{kl}}^{,l}{}_{,m}) \tag{30} $$

where $\Box$ denotes the d'Alembertian operator:

$$ \Box = -\eta^{lm}\frac{\partial^2}{\partial x^l \partial x^m} = \triangle - \frac{\partial^2}{c^2 \partial t^2} $$

To simplify the equation, we make use of the gauge freedom as observed in Eq.s (**??**) and (**??**)
.

We impose on h_{ik} the following additional condition–

$$ \psi_i^k = h_i^k - \frac{1}{2}\delta_i^k h $$

$$ \psi^k_{i,k} = 0 \tag{31} $$

This is known as the *harmonic gauge* or the *de Donder gauge*.

We again note that the above condtions still *do not* imply a preferred frame of reference. Substituting from Eq. (**??**) we can see that condition (31) allows for co-ordinate transformations as in Eq. (**??**) provided–

$$\Box \xi^i = 0 \tag{32}$$

The Eq. (31) gives us–

$$h^k_{i,k} = \frac{1}{2}\delta^k_i h_{,k} = \frac{1}{2}h_{,i}$$

Using this we get–

$$h^l_{m,k,l} - h_{,k,m} + h_{kl}{}^{,l}{}_{,m} = 0$$

And Eq. (**??**) reduces to –

$$R_{km} = \frac{1}{2}\Box h_{km} \tag{33}$$

For the case of vacuum we have

$$R_{km} = 0$$

and so Eq. (33) gives us the usual wave equation–

$$\Box h_{km} = 0 \tag{34}$$

3.5 2^{nd} order calculation

The wave equation obtained in the previous section is *linear*. As a result there is no self-interaction in the wave and so *one cannot hope to evolve an isotropic wave into an anisotropic one.* With this purpose in mind we here attempt to calculate the 2^{nd} order corrections to the above equations.

The second order Ricci tensor is given by Eq. (28)–

$$R_{km}{}^{(2)} = -\frac{1}{2} h^{ln} \left(h_{nm,kl} + h_{kl,nm} - h_{nl,km} - h_{km,nl} \right) \tag{35}$$

Thus the wave equation in vacuum reduces to –

$$\Box h_{km} = h^{ln} \left(h_{nm,kl} + h_{kl,nm} - h_{nl,km} - h_{km,nl} \right) \tag{36}$$

4 Perturbations

To analyse the phenomena of gravitational waves, a perturbation h_{ik} in the metric of spacetime g_{ik} over its "background" value $g_{ik}^{(0)}$ is considered as –

$$g_{ik} = g_{ik}{}^{(0)} + h_{ik} \tag{37}$$

It is chosen by convention to "raise" and "lower" indices using the "background" metric $g_{ik}^{(0)}$. That is - $h_k^m = g^{mi(0)} h_{ik}$, where –

$$g^{mi(0)} g_{ik}{}^{(0)} = \delta_k^m \tag{38}$$

The perturbations in the contravariant metric is evaluated by solving the equation –

$$g^{mi} g_{ik} = \delta_k^m \tag{39}$$

Let – $g^{mi} = g^{mi(0)} + f^{mi}$. Then substitution into above equation gives –

$$\left(g^{mi(0)} + f^{mi} \right) \left(g_{ik}{}^{(0)} + h_{ik} \right) = \delta_k^m$$

i.e.

$$h_k^m + f_k^m + f^{mi} h_{ik} = 0 \tag{40}$$

Now, upto first order in h we have –

$$f^{mk(1)} = -h^{mk}$$

Substituting this back in Equation(40) we find the next order as –

$$f^{mk^{(2)}} = h^{mi} h_i^k$$

Similarly we can iteratively find the higher orders of f as –

$$f^{mk^{(n)}} = -f^{mi^{(n-1)}} h_i^k \quad ; \quad f^{mk^{(1)}} = -h^{mk} \tag{41}$$

5 Christoffel Symbols

The Christoffel symbols in terms of the metric tensor are –

$$\Gamma^n_{km} = \frac{1}{2} g^{in} \left(g_{mi,k} + g_{ki,m} - g_{km,i} \right) \tag{42}$$

where the 'comma' (,) denotes partial derivatives.

Now, upto 2^{nd} order in the metric perturbations –

$$\Gamma^n_{km} = \frac{1}{2} \left(g^{in\,(0)} - h^{in} + h^i_p h^{pn} \right) \begin{pmatrix} g_{mi,k}{}^{(0)} + g_{ki,m}{}^{(0)} - g_{km,i}{}^{(0)} \\ h_{mi,k} + h_{ki,m} - h_{km,i} \end{pmatrix}$$

So the Christoffel symbols of different orders are –

$$\Gamma^n_{km}{}^{(0)} = \frac{1}{2} g^{in\,(0)} \left(g_{im,k}{}^{(0)} + g_{ik,m}{}^{(0)} - g_{km,i}{}^{(0)} \right) \tag{43}$$

$$\begin{aligned} \Gamma^n_{km}{}^{(1)} = & -\frac{1}{2} h^{in} \left(g_{im,k}{}^{(0)} + g_{ik,m}{}^{(0)} - g_{km,i}{}^{(0)} \right) \\ & + \frac{1}{2} g^{in\,(0)} \left(h_{im,k} + h_{ik,m} - h_{km,i} \right) \end{aligned} \tag{44}$$

$$\begin{aligned} \Gamma^n_{km}{}^{(2)} = & \frac{1}{2} h^i_p h^{pn} \left(g_{im,k}{}^{(0)} + g_{ik,m}{}^{(0)} - g_{km,i}{}^{(0)} \right) \\ & - \frac{1}{2} h^{in} \left(h_{im,k} + h_{ik,m} - h_{km,i} \right) \end{aligned} \tag{45}$$

6 Harmonic Gauge

To find specific solutions of the Einstein equation, the freedom in its general covariance is restricted by imposing certain additional conditions on the metric called *gauge conditions*.

The *harmonic gauge*, generally used in analysis of gravitational waves, is specified by the conditions –

$$H^n = g^{km}\Gamma^n_{km} = 0 \tag{46}$$

We now write these conditions for different orders of h –

$$H^{n\,(0)} = g^{km\,(0)}\Gamma^{n\;(0)}_{km}$$

$$= \frac{1}{2}g^{km\,(0)}g^{in\,(0)}\left(g_{im,k}{}^{(0)} + g_{ik,m}{}^{(0)} - g_{km,i}{}^{(0)}\right) \tag{47}$$

$$H^{n\,(1)} = g^{km\,(0)}\Gamma^{n\;(1)}_{km} + g^{km\,(1)}\Gamma^{n\;(0)}_{km}$$

$$= -\frac{1}{2}\left(g^{in\,(0)}h^{km} + g^{km\,(0)}h^{in}\right)\left(g_{im,k}{}^{(0)} + g_{ik,m}{}^{(0)} - g_{km,i}{}^{(0)}\right)$$

$$+ \frac{1}{2}g^{km\,(0)}g^{in\,(0)}\left(h_{im,k} + h_{ik,m} - h_{km,i}\right) \tag{48}$$

$$H^{n\,(2)} = g^{km\,(0)}\Gamma^{n\;(2)}_{km} + g^{km\,(2)}\Gamma^{n\;(0)}_{km} + g^{km\,(1)}\Gamma^{n\;(1)}_{km}$$

$$= \frac{1}{2}\left(g^{km\,(0)}h^i_p h^{pn} + g^{in\,(0)}h^k_p h^{pm} + h^{km}h^{in}\right)\left(g_{im,k}{}^{(0)} + g_{ik,m}{}^{(0)} - g_{km,i}{}^{(0)}\right)$$

$$- \frac{1}{2}\left(g^{in\,(0)}h^{km} + g^{km\,(0)}h^{in}\right)\left(h_{im,k} + h_{ik,m} - h_{km,i}\right) \tag{49}$$

7 Ricci Tensor

The Riemann tensor is given as –

$$R_{iklm} = \frac{1}{2}\left(g_{kl,mi} + g_{mi,kl} - g_{il,km} - g_{km,il}\right) + g_{jp}\left(\Gamma^j_{kl}\Gamma^p_{mi} - \Gamma^j_{km}\Gamma^p_{il}\right) \tag{50}$$

The Ricci tensor is thus –

$$R_{km} = R^l{}_{klm} = \frac{1}{2}g^{ln}\left(g_{mn,kl} + g_{kl,mn} - g_{nl,km} - g_{km,nl}\right) + g^{ln}g_{jp}\left(\Gamma^j_{kl}\Gamma^p_{mn} - \Gamma^j_{km}\Gamma^p_{nl}\right) \tag{51}$$

Now in the case of a Minkowskian background, we have ${g_{ik}}^{(0)} = \eta_{ik}$. So ${R_{km}}^{(0)} = 0$ and for first order we have –

$$\begin{aligned} {R_{km}}^{(1)} &= \frac{1}{2}\eta^{ln}\left(h_{kl,mn} + h_{mn,kl} - h_{nl,km} - h_{km,nl}\right) \\ &= \frac{1}{2}\left(h^n_{k,nm} + h^l_{m,lk} - h_{,km} + \Box h_{km}\right) \end{aligned} \tag{52}$$

Also from Equation (48) the harmonic gauge condition becomes –

$$h^l_{k,l} - \frac{1}{2}h_{,k} = 0 \tag{53}$$

and thus we have –

$${R_{km}}^{(1)} = \frac{1}{2}\Box h_{km} \tag{54}$$

which in the first order approximation reduces to the ordinary wave equation.

Also the second order Ricci tensor is –

$$\begin{aligned} &-\frac{1}{2}h^{ln}\left(h_{kl,mn} + h_{mn,kl} - h_{km,nl} - h_{nl,km}\right) \\ &+ \frac{1}{4}\eta^{ln}\eta_{jp}\left[\begin{array}{c} \left(h^j_{k,l} + h^j_{l,k} - {h_{kl}}^{,j}\right)\left(h^p_{m,n} + h^p_{n,m} - {h_{mn}}^{,p}\right) \\ -\left(h^j_{k,m} + h^j_{m,k} - {h_{km}}^{,j}\right)\left(h^p_{l,n} + h^p_{n,l} - {h_{nl}}^{,p}\right) \end{array}\right] \end{aligned} \tag{55}$$

Using the harmonic gauge as in Equation.(53) we get –

$$\eta^{ln}\left(h^{p}_{l,n}+h^{p}_{n,l}-h_{nl}{}^{,p}\right)=0$$

and the second term reduces to –

$$\frac{1}{4}\left(h_{pk}{}^{,n}+h^{n}_{p,k}-h^{n}_{k,p}\right)\left(h^{p}_{m,n}+h^{p}_{n,m}-h_{mn}{}^{,p}\right)$$

Now we use the condition of Ricci-flatness $R_{km}=0$ and thus we get the nonlinear wave equation –

$$\Box h_{km}=h^{ln}\left(h_{kl,mn}+h_{mn,kl}-h_{km,nl}-h_{nl,km}\right)$$

$$-\frac{1}{2}\left(h_{pk}{}^{,n}+h^{n}_{p,k}-h^{n}_{k,p}\right)\left(h^{p}_{m,n}+h^{p}_{n,m}-h_{mn}{}^{,p}\right) \tag{56}$$

8 The Wave Equations

Now lets us assume that the metric perturbation h takes a form similar to the linear solution of a transverse wave traveling along the Z - axis. So we can write h_{ik} as –

$$h_{ik} = \begin{pmatrix} k & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{57}$$

where a, b, k are function of x^k.

For the case of Minkowski background, the zeroth order harmonic condition is satisfied identically. We thus, evaluate the harmonic gauge conditions of 1^{st} order –

$$\begin{aligned} H^{n(1)} &= h^{0n}_{,0} + h^{1n}_{,1} + h^{2n}_{,2} - \frac{1}{2}\left(h^{0,n}_{0} + h^{1,n}_{1} + h^{2,n}_{2}\right) \\ &= h^{0n}_{,0} + h^{1n}_{,1} + h^{2n}_{,2} - \frac{1}{2} h^{0,n}_{0} \end{aligned}$$

Thus for different values $n = 0, 1, 2, 3$, we get –

$$H^{0(1)} = \frac{1}{2}\frac{\partial k}{\partial t} = 0 \tag{58}$$

$$H^{1(1)} = \frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} + \frac{1}{2}\frac{\partial k}{\partial x} = 0 \tag{59}$$

$$H^{2(1)} = \frac{\partial b}{\partial x} - \frac{\partial a}{\partial y} + \frac{1}{2}\frac{\partial k}{\partial y} = 0 \tag{60}$$

$$H^{3(1)} = \frac{1}{2}\frac{\partial k}{\partial z} = 0 \tag{61}$$

These equations will help simplify the wave equation further.

The wave equation can now be written for each component of h_{ik}. For $\mu = 1, 2, 3$, we have $h_{0\mu} = 0$, and using the first and last harmonic conditions we have –

$$0 = h^{11}\left(h_{\mu 1,01} - h_{11,0\mu}\right) + h^{12}\left(h_{\mu 2,01} - h_{12,0\mu}\right) + h^{21}\left(h_{\mu 1,02} - h_{21,0\mu}\right) + h^{22}\left(h_{\mu 2,02} - h_{22,0\mu}\right)$$

$$-\frac{1}{2}\left(\begin{array}{l} h^{;1}_{00}\left(-h_{\mu 1}{}^{,0}\right) + h^{;2}_{00}\left(-h_{\mu 2}{}^{,0}\right) - h^{0}_{0,1}\left(h^{1}_{\mu,0}\right) - h^{0}_{0,2}\left(h^{2}_{\mu,0}\right) \\ +h^{1}_{1,0}\left(h^{1}_{\mu,1} + h^{1}_{1,\mu} - h_{\mu 1}{}^{,1}\right) + h^{1}_{2,0}\left(h^{2}_{\mu,1} + h^{2}_{1,\mu} - h_{\mu 1}{}^{,2}\right) \\ +h^{2}_{1,0}\left(h^{1}_{\mu,2} + h^{1}_{2,\mu} - h_{\mu 2}{}^{,1}\right) + h^{2}_{2,0}\left(h^{2}_{\mu,2} + h^{2}_{2,\mu} - h_{\mu 2}{}^{,2}\right) \end{array}\right)$$

Now for $\mu = 1$, we get –

$$b\frac{\partial}{\partial t}\left(\frac{\partial a}{\partial y} - \frac{\partial b}{\partial x}\right) - a\frac{\partial}{\partial t}\left(\frac{\partial a}{\partial x} + \frac{\partial b}{\partial y}\right)$$

$$-\left(\frac{\partial k}{\partial x}\frac{\partial a}{\partial t} + \frac{\partial k}{\partial y}\frac{\partial b}{\partial t}\right) - \left(\frac{\partial a}{\partial x}\frac{\partial a}{\partial t} + \frac{\partial b}{\partial x}\frac{\partial b}{\partial t}\right) = 0$$

On simplification using the harmonic conditions, we have –

$$\frac{\partial a}{\partial t}\left(2\frac{\partial b}{\partial y} + \frac{\partial a}{\partial x}\right) = \frac{\partial b}{\partial t}\left(2\frac{\partial a}{\partial y} - \frac{\partial b}{\partial x}\right) \tag{62}$$

Similarly for $\mu = 2$ we have–

$$\frac{\partial b}{\partial t}\left(2\frac{\partial a}{\partial x} + \frac{\partial b}{\partial y}\right) = \frac{\partial a}{\partial t}\left(2\frac{\partial b}{\partial x} - \frac{\partial a}{\partial y}\right) \tag{63}$$

and for $\mu = 3$ –

$$a\frac{\partial^2 a}{\partial t\partial z} + b\frac{\partial^2 b}{\partial t\partial z} = -\frac{1}{2}\left(\frac{\partial a}{\partial t}\frac{\partial a}{\partial z} + \frac{\partial b}{\partial t}\frac{\partial b}{\partial z}\right) \tag{64}$$

For $\mu = 1, 2, 3$, $h_{3\mu} = 0$ so we have –

$$0 = h^{11}\left(h_{\mu 1,31} - h_{11,3\mu}\right) + h^{12}\left(h_{\mu 2,31} - h_{21,3\mu}\right) + h^{21}\left(h_{\mu 1,32} - h_{12,3\mu}\right) + h^{22}\left(h_{\mu 2,32} - h_{22,3\mu}\right)$$

$$-\frac{1}{2}\left(\begin{array}{c} h^1_{1,3}\left(h^1_{\mu,1} + h^1_{1,\mu} - h^{;1}_{\mu 1}\right) + h^1_{2,3}\left(h^2_{\mu,1} + h^2_{1,\mu} - h^{;2}_{\mu 1}\right) \\ h^2_{1,3}\left(h^1_{\mu,2} + h^1_{2,\mu} - h^{;1}_{\mu 2}\right) + h^2_{2,3}\left(h^2_{\mu,2} + h^2_{2,\mu} - h^{;2}_{\mu 2}\right) \end{array}\right)$$

Now for $\mu = 1$

$$b\frac{\partial}{\partial z}\left(\frac{\partial a}{\partial y}-\frac{\partial b}{\partial x}\right)-a\frac{\partial}{\partial z}\left(\frac{\partial a}{\partial x}+\frac{\partial b}{\partial y}\right)-\frac{\partial a}{\partial z}\frac{\partial a}{\partial x}-\frac{\partial b}{\partial z}\frac{\partial b}{\partial x}=0$$

Using the harmonic conditions we have –

$$\frac{\partial a}{\partial z}\frac{\partial a}{\partial x}=-\frac{\partial b}{\partial z}\frac{\partial b}{\partial x} \tag{65}$$

Similarly for $\mu = 2$

$$\frac{\partial a}{\partial z}\frac{\partial a}{\partial y}=-\frac{\partial b}{\partial z}\frac{\partial b}{\partial y} \tag{66}$$

and $\mu = 3$

$$a\frac{\partial^2 a}{\partial z^2}+b\frac{\partial^2 b}{\partial z^2}=-\frac{1}{2}\left[\left(\frac{\partial a}{\partial z}\right)^2+\left(\frac{\partial b}{\partial z}\right)^2\right] \tag{67}$$

Similarly writing the components explicitly we get for $h_{11} = a$ –

$$\begin{aligned}\Box a = &-k\left(\frac{\partial^2 k}{\partial x^2}+\frac{\partial^2 a}{\partial t^2}\right)-a\left(2\frac{\partial^2 b}{\partial x\partial y}+\frac{\partial^2 a}{\partial x^2}-\frac{\partial^2 a}{\partial y^2}\right)\\ &-\left[\left(\frac{\partial a}{\partial x}\right)^2+\left(\frac{\partial a}{\partial y}\right)^2+\left(\frac{\partial a}{\partial z}\right)^2\right]+\left(\frac{\partial b}{\partial x}\right)\left(\frac{\partial k}{\partial y}\right)-\left(\frac{\partial b}{\partial z}\right)^2\\ &+\left[\left(\frac{\partial a}{\partial t}\right)^2+\left(\frac{\partial b}{\partial t}\right)^2\right]-\frac{1}{2}\left(\frac{\partial k}{\partial x}\right)^2\end{aligned} \tag{68}$$

For $h_{12} = b$ –

$$\begin{aligned}\Box b = &-k\left(\frac{\partial^2 k}{\partial x\partial y}+\frac{\partial^2 b}{\partial t^2}\right)-b\left(2\frac{\partial^2 b}{\partial x\partial y}+\frac{\partial^2 a}{\partial x^2}-\frac{\partial^2 a}{\partial y^2}\right)\\ &-2\left(\frac{\partial a}{\partial x}\right)\left(\frac{\partial a}{\partial y}\right)+\left(\frac{\partial a}{\partial x}\right)\left(\frac{\partial b}{\partial x}\right)-\left(\frac{\partial a}{\partial y}\right)\left(\frac{\partial b}{\partial y}\right)-\frac{1}{2}\left(\frac{\partial k}{\partial x}\right)\left(\frac{\partial k}{\partial y}\right)\end{aligned} \tag{69}$$

For $h_{21} = b$ we get the same equation as above as is to be expected.

For $h_{22} = -a$ –

$$\Box a = -k\left(\frac{\partial^2 k}{\partial y^2} + \frac{\partial^2 a}{\partial t^2}\right) - a\left(2\frac{\partial^2 b}{\partial x \partial y} + \frac{\partial^2 a}{\partial x^2} - \frac{\partial^2 a}{\partial y^2}\right)$$

$$+\left[\left(\frac{\partial a}{\partial x}\right)^2 + \left(\frac{\partial a}{\partial y}\right)^2 + \left(\frac{\partial a}{\partial z}\right)^2\right] - \left(\frac{\partial b}{\partial y}\right)\left(\frac{\partial k}{\partial x}\right) + \left(\frac{\partial b}{\partial z}\right)^2$$

$$-\left[\left(\frac{\partial a}{\partial t}\right)^2 + \left(\frac{\partial b}{\partial t}\right)^2\right] + \frac{1}{2}\left(\frac{\partial k}{\partial y}\right)^2 \tag{70}$$

Finally for $h_{00} = k$ we get —

$$\Box k = -a\left(\frac{\partial^2 k}{\partial x^2} - \frac{\partial^2 k}{\partial y^2}\right) - 2b\frac{\partial^2 k}{\partial x \partial y} + \left(\frac{\partial k}{\partial x}\right)^2 + \left(\frac{\partial k}{\partial y}\right)^2$$

$$-2\left(a\frac{\partial^2 a}{\partial t^2} + b\frac{\partial^2 b}{\partial t^2}\right) - \left[\left(\frac{\partial a}{\partial t}\right)^2 + \left(\frac{\partial b}{\partial t}\right)^2\right] \tag{71}$$

9 Solution

We can find a solution by assuming that $k = constant$ and $a = Au(z,t)$ and $b = Bu(z,t)$ where $A, B = constants$. For this case the most of the equations identically vanish and we are left with –

$$\Box u = -k\frac{\partial^2 u}{\partial t^2} \quad ; \quad \left(\frac{\partial u}{\partial t}\right)^2 = \left(\frac{\partial u}{\partial z}\right)^2$$

$$u\frac{\partial^2 u}{\partial t^2} = -\frac{1}{2}\left(\frac{\partial u}{\partial t}\right)^2 \quad ; \quad u\frac{\partial^2 u}{\partial z^2} = -\frac{1}{2}\left(\frac{\partial u}{\partial z}\right)^2 \quad ; \quad u\frac{\partial^2 u}{\partial t \partial z} = -\frac{1}{2}\left(\frac{\partial u}{\partial t}\right)\left(\frac{\partial u}{\partial z}\right)$$

The second equation from these has two independent solutions –

$$u(z,t) = u_+(z+t) \quad ; \quad u(z,t) = u_-(z-t)$$

Using these the LHS of the first equation is identically zero giving $k = 0$ and the other three equations all reduce to –

$$uu'' + \frac{1}{2}u'^2 = 0 \tag{72}$$

where the prime " $'$ " denotes differentiation by $\lambda_\pm = z \pm t$ for $u_\pm$ respectively.

To solve this we write $u' = v$ and thus $u'' = v\frac{dv}{du}$ and get –

$$uv\frac{dv}{du} + \frac{1}{2}v^2 = 0$$

Assuming the $v \neq 0$ –

$$u\frac{dv}{du} = -\frac{1}{2}v$$

which is readily integrated to give –

$$v = Cu^{-1/2}$$

where C is the integration constant.

Further using $v = \frac{du}{d\lambda}$ we get the solution as —

$$u_\pm(z,t) = C\left(z \pm t + \delta\right)^{2/3} \tag{73}$$

This can be written in the spherical polar coordinates as –

$$u_\pm(t,r,\theta,\phi) = C\left(rcos\theta \pm t + \delta\right)^{2/3} \tag{74}$$

As any other function this can be expanded in terms of the spherical harmonics to give the various multipole moments for this wave.

$$u_\pm(t,r,\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{+l} C_l j_l\left(kr\right) Y_l^m\left(\theta,\phi\right)$$

where the $j_l(x)$ represents the spherical Bessel functions of order l and $k = \frac{2\pi}{\lambda}$ is the wave number of the gravitational perturbation.

This analysis assumes the existence of a flat background geometry. This can be assumed to be valid in a FRW expanding universe provided that the wavelength of the gravitational waves is small enough (equivalently large k). Thus the above analysis represents the large k limit of the full-calculation.

To do the full calculation in the FRW background a computational code was written in Mathematica and the Ricci tensor computed for a second order perturbation in FRW space-time.

l	l(l+1)$\lvert C_l\rvert^2$	C_l
10	900	2.861
100	2500	0.497
200	5800	0.380
300	3500	0.197
420	1800	0.101
540	2500	0.093
700	1800	0.004
820	2000	0.054

10 Fitting WMAP data

To model the anisotropy in the CMBR we can expand the fluctuations in terms of the orthogonal spherical harmonic functions as –

$$\sum_{l=0}^{\infty}\sum_{m=-l}^{+l} C_l Y_l^m\left(\theta,\phi\right)$$

where the Y_l^m can be written in terms of the Associated Legendre Polynomials as –

$$Y_l^m\left(\theta,\phi\right) = (-1)^{\frac{m+|m|}{2}}\sqrt{\frac{2l+1}{4\pi}\frac{(l-|m|)!}{l+|m|}!}P_l^m\left(cos\theta\right)e^{\iota m\phi}$$

To get the coefficients C_l we can sample the WMAP plot at various points of interest and generate a fit to the curve.

One such fit produced is –

$$l(l+1)|C_l|^2 = 13000exp(-\frac{l}{300})J_4(\frac{l}{40}) + 2500$$

where $J_4(x)$ represents the Bessel function of order 4.

Thus the form of C_l obtained is –

$$C_l = \left[\frac{13000exp(-\frac{l}{300})J_4(\frac{l}{40}) + 2500}{l(l+1)}\right]^{1/2}$$

Using this form for C_l the anisotropy can be plotted as 3D polar plot to get a picture of the various intricate patterns in the gravitational wave spectrum.

Such a plot was made for a sample of 8 points taken from the WMAP plot.

This represents only a coarse-grain model. For an accurate model more number of data points would have to be sampled resulting in a better fit to actual data.

Though a linear analysis would have given a ripple spectrum but there is an explicit mechanism to generate higher-l modes in a second order calculation.

The nonlinear coupling, through product of first derivatives, in the gravitational perturbation can result in generation of higher harmonic modes in the perturbation. Beginning with lower modes

$l = 0, 1, 2$ one can generate the observed anisotropy caused due to the nonlinear coupling of the wave as seen in the nonlinear equation.

This nonlinear coupling can transfer energy from lower l-modes to higher l-modes. This results in an anisotropic wrinkled space-time. This can be linked to the density perturbations as observed in CMBR and WMAP as done below.

11 Kinematics of distribution of matter through a simple model

11.1 Diffusion equation model

In our context the word diffusion applies to motion of particles of matter matter may be normal matter,dark matter or dark energy. Let us try to solve the diffusion problem in one dimensions. the method for higher dimensions follows similarlyDivide the space into thin parts.The process of diffusion can be considered effectively as the problem of random walk which deals with the statistical progress of a particle equally likely to move forwards or backwards. At any instant of time t half the particles in a part at the position x diffuse to the part at $x + \Delta x$ and half the particles diffuse to part at $x - \Delta x$.The variables x and t being continuous require a population density function n(x,t);its meaning is that n(x,t)$\Delta x$is the population occupying a short section Δx. Hence we get

$$\frac{1}{2}n(x + \Delta x, t) + \frac{1}{2}n(x - \Delta x, t) = n(x, t + \Delta t)$$

Subtracting n(x,t) from both sides, it is supposed that the difference is very small compared to n(x,t) itself, we get

$$\Rightarrow \frac{1}{2}[n(x + \Delta x, t) - n(x, t)] - \frac{1}{2}[n(x, t) - n(x - \Delta x, t)] = n(x, t + \Delta t) - n(x, t)$$

$$\Rightarrow \frac{1}{2}\left(\left(\frac{\partial n}{\partial x}\right)_x - \left(\frac{\partial n}{\partial x}\right)_{x-\Delta x}\right)\Delta x = \left(\frac{\partial n}{\partial t}\right)\Delta t$$

$$\Rightarrow \frac{1}{2}(\Delta x)^2\frac{\partial^2 n}{\partial x^2} = \left(\frac{\partial n}{\partial t}\right)\Delta t$$

The final step is to assume that, with a large number of particles, Δx and Δt can be reduced indefinitely (which makes the above approximations exact) but keeping the ratio $D = \frac{\frac{1}{2}(\Delta x)^2}{\Delta t}$ constant. Thus we have the diffusion equation as

$$D\frac{\partial^2 n}{\partial x^2} = \left(\frac{\partial n}{\partial t}\right)$$

which means that the random diffusing process which we envisage does indeed lead to the diffusion equation. A special solution of diffusion equation is the Gaussian which is

$$n(x, t) = (\pi D t)^{\frac{-1}{2}} e^{\left(\frac{-x^2}{4Dt}\right)}$$

The above equation gives the distribution of number of particles with respect to position at a given time t. If there was very distribution at x_0 at time t_0 then we write the Gaussian as

$$n(x,t) = (\pi D(t-t_0))^{\frac{-1}{2}} e^{\left(\frac{-(x-x_0)^2}{4D(t-t_0}\right)} \tag{75}$$

As mentioned in the introduction we are trying to model the redistribution of matter in the Universe as a process of diffusion running backwards in time. Looked at the scale of the Universe the various points of accumulation of normal matter correspond to super-clusters. At the scale of super-clusters points of accumulation of normal matter correspond to clusters and the scale of clusters to galaxies. Firstly we explain the clustering and clumping of normal matter at the scale of the Universe.Similar arguments are valid at the scale of super-clusters,clusters and galaxies. Now if the process of clustering of normal matter is diffusion running backwards in time and equation 75 is a solution of diffusion equation and t_0 denotes the present time then x_0 would be the position of a typical super-cluster. But equation 75 would mean that all the matter is concentrated at the point x_0 at present time. This would mean that a super-cluster has no size which is wrong. Therefore we add a $-\sigma^2$ term to equation 75 where σ is the standard deviation which is an estimate of the size of super-cluster. For an Gaussian σ corresponds to half of the width of the Gaussian at half maximum which is 65% of the total width of the Gaussian. For a Gaussian representing a super-cluster the radius of super cluster is roughly 3σ as $3\sigma = 95\%$ of all matter is within the cluster entity of this radius.

Now having modeled the distribution of normal matter through a diffusion equation we can account for the redistribution of dark-matter and dark-energy also in a similar way. The most important point to be noted is that the Gaussian for different forms of matter will have different values of Diffusion coefficient D and different values for standard deviation σ. This is the main reason why different forms of matter differentiate among each other and ended up at different places in the Universe. 1-d as well as 2-d plots were made using Matlab of density of normal matter v/s position for various times at the scale of Universe. From these figures one can see how a uniform distribution of normal matter ultimately led to the formation of super-clusters with sharp densities. The formation of clusters within a super cluster and of galaxies within a cluster can be explained similarly. Note that in the real diffusion process of a gas the series of pictures would be from sharp to wide gaussians. The radius of the Universe is $4000 mega - parsecs \sim 10^{26} m$.In the 1-d plot 1 unit=10 mega-parsecs and in 2-d plot 1 unit=100 mega-parsecs were used.

We now draw a rough analogy between the distribution of various forms of matter in the Universe and condensation of air. The rough mass composition of air in our atmosphere is 70% of N_2 27% of O_2,1% of CO_2,0.5% of H_2O and 0.1 % of CO. As the temperature keeps dropping different gases start to condense. The process begins with the condensation of water-vapor. At various points in air one finds formation of droplets of water at different places as we go closer to 0^0C. Condensation is nothing but bringing water molecules which were far closer to each other until sufficient accumulation takes place as the phase changes from gas to liquid. As the temperature goes down further to -40^0C the CO_2 in the atmosphere also condenses and similarly various gases condense at various stages. Now the redistribution of various forms of matter in the Universe is analogous to condensation of air in the sense that various forms of matter accumulate at different points in space.Note that the mass composition of our Universe is 69.7475% of Dark Energy present in voids,29.2475% of Dark matter out which 2.5% is in galaxy halos and the rest is in voids,0.5% of normal matter and .005 % of radiation present everywhere in the Universe.

Name	Number	Size	Mass
Universe	1	order of 10^{26} meters (around 4000 mega parsecs 10^9 light years	10^{20} solar masses
Supercluster	roughly 1000	400 mega parsecs	10^{17} solar masses
Cluster	256000	40 mega parsecs	10^{15} solar masses [1]
Galaxy	10^{11} galaxies in the Universe	4 mega parsecs	10^9 solar masses

Now we try to establish a criterion to find out whether or not a given set of cluster of galaxies form a super-cluster.Let the size of each grid be 1 unit $\times$ 1 unit.Now from the figure one can make out various patterns of super-cluster of cluster of galaxies.But can we try to obtain a quantitative result in order to define a super-cluster.For this purpose consider any pattern from the figure.For example we consider the pattern of clusters with coordinates:

(28,30);(27,32);(30,32);(26,34);(29,34);(31,36);(29,37);(31,38);(29,40)

Since different clusters are separated from each other by different distances we need to obtain a quantity which represents the average value of all distances.Now if the evaluated mean is less than some fixed value then the given group of clusters form a super cluster.If (x_i, y_i) and (x_j, y_j) are the coordinates of the ith and jth cluster then the following quantities may be evaluated for a set of n clusters :

1.

$$\left(\frac{\sum_{i=1}^{n}\sum_{j=i+1}^{n}[(x_i-x_j)^2+(y_i-y_j)^2]}{n}\right)^{\frac{1}{2}} = 10.203$$

2.

$$\left(\frac{\sum_{i=1}^{n}\sum_{j=i+1}^{n}[(x_i-x_j)^2+(y_i-y_j)^2]}{{}^{n}C_2}\right)^{\frac{1}{2}} = 5.102$$

3.

$$\left(\frac{\sum_{i=1}^{n}\sum_{j=i+1}^{n}[(x_i - x_j)^2 + (y_i - y_j)^2]^{\frac{1}{2}}}{n}\right) = 20.403$$

4.

$$\left(\frac{\sum_{i=1}^{n}\sum_{j=i+1}^{n}[(x_i - x_j)^2 + (y_i - y_j)^2]^{\frac{1}{2}}}{{}^{n}C_2}\right) = 5.101$$

Sl.no	positions of cluster	Δx i.e, x separation	Δy i.e, y separation	$(\Delta x)^2 + (\Delta y)^2$	$((\Delta x)^2 + (\Delta y)^2)^{\frac{1}{2}}$
1	(28,30)(27,32)	1	2	5	$\sqrt{5}$
2	(28,30)(30,32)	2	2	8	$2\sqrt{2}$
3	(28,30)(26,34)	2	4	20	$2\sqrt{5}$
4	(28,30)(29,34)	1	4	17	$\sqrt{17}$
5	(28,30)(31,36)	3	6	45	$3\sqrt{5}$
6	(28,30)(29,37)	1	7	50	$5\sqrt{2}$
7	(28,30)(31,38)	3	8	73	$\sqrt{73}$
8	(28,30)(29,40)	1	10	101	$\sqrt{101}$
9	(27,32)(30,32)	3	0	9	3
10	(27,32)(26,34)	1	2	5	$2\sqrt{5}$
11	(27,32)(29,34)	4	4	8	$2\sqrt{2}$
12	(27,32)(31,36)	4	4	32	$4\sqrt{2}$
13	(27,32)(29,37)	2	5	29	$\sqrt{29}$
14	(27,32)(31,38)	4	6	52	$\sqrt{52}$
15	(27,32)(29,40)	2	8	68	$2\sqrt{17}$
16	(30,32)(26,34)	4	2	20	$2\sqrt{5}$
17	(30,32)(29,34)	1	2	5	$\sqrt{5}$
18	(30,32)(31,36)	1	4	17	$\sqrt{17}$
19	(30,32)(29,37)	1	5	26	$\sqrt{26}$
20	(30,32)(31,38)	1	6	37	$\sqrt{37}$
21	(30,32)(29,40)	1	8	65	$2\sqrt{65}$
22	(26,34)(29,34)	3	0	9	3
23	(26,34)(31,36)	2	2	8	$2\sqrt{2}$
24	(26,34)(29,37)	3	3	18	$3\sqrt{2}$
25	(26,34)(31,38)	5	4	41	$2\sqrt{41}$
26	(26,34)(29,40)	3	6	45	$3\sqrt{5}$
27	(29,34)(31,36)	2	2	8	$2\sqrt{2}$
28	(29,34)(29,37)	0	3	9	3
29	(29,34)(31,38)	2	4	20	$2\sqrt{5}$
30	(29,34)(29,40)	0	6	36	6
31	(31,36)(29,37)	2	1	5	$\sqrt{5}$
32	(31,36)(31,38)	0	2	4	2
33	(31,36)(29,40)	2	4	20	$2\sqrt{5}$
34	(29,37)(31,38)	2	1	5	$\sqrt{5}$
35	(29,37)(29,40)	0	3	9	3
36	(31,38)(29,40)	2	2	8	$2\sqrt{2}$

12 Density perturbation using FRW metric model

We discuss the model based on the Friedman-Robertson-Walker (FRW) model. The FRW metric is given by

$$ds^2 = c^2 dt^2 - a^2(t)(dr^2 + r^2(d\theta)^2 + r^2 \sin^2\theta (d\phi)^2)$$

Einstein's field equation :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \left(\frac{8\pi G}{c^4}\right) T_{\mu\nu}$$

where $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the Einstein tensor,R is the Ricci scalar and $T_{\mu\nu}$ is the stress-energy tensor.With the above two equations get at the following two equations [2]

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2 + kc^2}{a^2} = \left(\frac{8\pi G}{c^4}\right) T_3^3$$

$$\frac{\dot{a}^2 + kc^2}{a^2} = \left(\frac{8\pi G\rho_0}{3c^2}\right) T_0^0$$

where k is the curvature constant.

Assuming that the Universe is filled with Pressure less dust then the stress energy tensor $T_{\mu\nu}$ becomes

$$\begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Hence assuming a pressureless dust-dominated Universe and taking the curvature constant to be 0 in the above two equations we get the following two equations.

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = 0 \tag{76}$$

$$\frac{\dot{a}^2}{a^2} = \left(\frac{8\pi G\rho_0}{3c^4}\right) \frac{a_0^3}{a^3} \tag{77}$$

Solving equation 77 we get

$$\dot{a}^2 = \left(\frac{8\pi G\rho_0}{3c^4}\right) \frac{a_0^3}{a}$$

If t_0 represents the present time then

$$\left(\frac{\dot{a}}{a}\right)_{t_0} = H_0$$

$$\rho_0 \equiv \frac{3H_0^2 c^4}{8\pi G} \tag{78}$$

We now solve equation 77 by rewriting as

$$\dot{a}^2 = H_0^2 \frac{a_0^3}{a}$$

[2] Reference 1 Chapter 4

$$\Rightarrow \dot{a} = H_0 \frac{a_0^{\frac{3}{2}}}{a^{\frac{1}{2}}}$$

$$\Rightarrow \int a^{\frac{1}{2}}\, da = \int H_0 a_0^{\frac{3}{2}}\, dt$$

$$\Rightarrow a^{\frac{3}{2}} = \frac{3}{2} H_0 a_0^{\frac{3}{2}} t$$

Assuming a=0 at t=0 the arbitrary constant that arises out of integration is set to 0.

Putting

$$H_0 = \frac{2}{3t_0} \tag{79}$$

$$\Rightarrow a^{\frac{3}{2}} = \frac{3}{2}\frac{2}{3t_0} a_0^{\frac{3}{2}} t$$

Putting $H_0 = \frac{2}{3t_0}$ where t_0 is the present time.

$$\Rightarrow a = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}} \tag{80}$$

One may as well check that this solution of a satisfies 76 by substituting $\dot{a} = \frac{2}{3} a_0 \left(\frac{t}{t_0}\right)^{\frac{-1}{3}} \frac{1}{t_0}$ and $\ddot{a} = \frac{-2}{9} a_0 \left(\frac{t}{t_0}\right)^{\frac{-4}{3}} (\frac{1}{t_0})^2$

As we are considering a model of Universe in which mass (M(t)) is conserved we have

$$\left(\frac{d(M(t))}{dt}\right) = 0$$

$$\Rightarrow \left(\frac{d(\rho(t)V(t))}{dt}\right) = 0$$

where $\rho(t)$and$V(t)$ are the density and volume of the Universe respectively.

$$\Rightarrow \rho(t)\left(\frac{d(V(t))}{dt}\right) + V(t)\left(\frac{d(\rho(t))}{dt}\right) = 0$$

$V(t) = bR^3$ where R is the radius of the Universe which is a function of time and b is a constant. Put $\rho(t) \equiv \rho$ then

$$3b\rho R^2 \left(\frac{d(R)}{dt}\right) + bR^3 \left(\frac{d(\rho)}{dt}\right) = 0$$

If $\frac{d(R)}{dt} \equiv \dot{R}$ and $\frac{d(\rho)}{dt} \equiv \dot{\rho}$ then

$$3\rho\dot{R} = -R\dot{\rho}$$

Since $\frac{\dot{R}}{R} = \frac{\dot{a}}{a}$

$$\frac{\dot{\rho}}{\rho} = \frac{-3\dot{R}}{R} = \frac{-3\dot{a}}{a}$$

Finally

$$\frac{\dot{\rho}}{\rho} = \frac{-3\dot{a}}{a} \tag{81}$$

$$\frac{\dot{\rho}}{\rho} = \frac{-3\dot{R}}{R} = \frac{-3\dot{a}}{a}$$

Conservation of mass would imply that

$$\rho R^3 = \rho_0 R_0^3$$

Using

$$\frac{a}{a_0} = \frac{R}{R_0}$$

$$\rho \left(\frac{a}{a_0}\right)^3 = \rho_0$$

Substituting 'a' as a function of time from equation 80 we get

$$\rho = \frac{\rho_0 t_0^2}{t^2} \tag{82}$$

Till now we have discussed equations and solutions which are valid provided our Universe is isotropic with respect to space i.e, if the scale factor 'a' were function of time alone. The FRW model is conventionally done in spherical-polar co-ordinates. Now if isotropy and homogeneity is broken we modify a(t) so that $\frac{\partial a}{\partial \theta} \neq 0$ and $\frac{\partial a}{\partial \phi} \neq 0$.For a 3 sphere we write down $a(t, \theta, \phi)$.

$$a(t, \theta, \phi) = a_1(t) + a_2(t, \theta, \phi) \tag{83}$$

Similarly for the matter part we write down $\rho(t, \theta, \phi)$

$$\rho(t, \theta, \phi) = \rho_1(t) + \rho_2(t, \theta, \phi) \tag{84}$$

If P represents the pressure and T the temperature at WMAP time we have $\frac{\Delta \rho}{\rho} \simeq \frac{\Delta T}{T} \simeq \frac{\Delta P}{P} \simeq 10^{-5}$ which increases with subsequent time. Hence $\frac{\rho_2}{\rho} \simeq \frac{a_2}{a} \simeq 10^{-5}$ at WMAP time ($10^{13} seconds$.So in the equations 76,77 and 81 we include the effect of a_2 and try to obtain the new set of equations with $\frac{a_2}{a} \ll 1$, $\frac{\rho_2}{\rho} \ll 1$.

The detailed structure formation on the Universe as seen at z (doppler shift) =0 to 1 with voids,cluster of galaxies and various kinds of fluctuations requires super computing models to put observed data from HST, SDSS etc which is outside the scope of this project. We solve for density fluctuations from WMAP i.e,z=1000 till z=5 and density fluctuations from z=5 to z=1. Recalling that

$$1 + z = \frac{a(t_0)}{a(t)} = \left(\frac{t_0}{t}\right)^{\frac{2}{3}}$$

Now combining 76 and 77 we get

$$\frac{2\ddot{a}}{a} + \left(\frac{8\pi G \rho_0}{3c^4}\right)\frac{a_0^3}{a^3} = 0 \tag{85}$$

Introducing inhomogeneity and anisotropy in $a \equiv a(t, \theta, \phi)$ as explained above we have

$$\frac{2(\ddot{a_1} + \ddot{a_2})}{a_1 + a_2} + \frac{8\pi G\rho_0 a_0^3}{3c^4(a_1 + a_2)^3} = 0$$

Note that $a_1 \equiv a_1(t)$ and $a_2 \equiv a_2(t, \theta, \phi)$

$$\Rightarrow \frac{2(\ddot{a_1} + \ddot{a_2})}{a_1}\left(1 + \frac{a_2}{a_1}\right)^{-1} + \frac{8\pi G\rho_0 a_0^3}{3c^4 a_1^3}\left(1 + \frac{a_2}{a_1}\right)^{-3} = 0$$

In the binomial expansion $\left(1 + \frac{a_2}{a_1}\right)^{-1}$ and $\left(1 + \frac{a_2}{a_1}\right)^{-3}$ we neglect the terms having the power of a_2 greater than 1

$$\Rightarrow \frac{2(\ddot{a_1} + \ddot{a_2})}{a_1}\left(1 - \frac{a_2}{a_1}\right) + \frac{8\pi G\rho_0 a_0^3}{3c^4 a_1^3}\left(1 - \frac{3a_2}{a_1}\right) = 0$$

$$\textit{Approximating } \frac{2\ddot{a_2}a_2}{a_1 a_1} \textit{ to } 0 \textit{ we get}$$

$$\frac{2\ddot{a_1}}{a_1} - \frac{2\ddot{a_1}a_2}{a_1 a_1} + \frac{2\ddot{a_2}}{a_1} + \frac{8\pi G\rho_0 a_0^3}{3c^4 a_1^3} - \frac{3a_2}{a_1}\left(\frac{8\pi G\rho_0 a_0^3}{3c^4 a_1^3}\right) = 0$$

Using equation 85 we have

$$\frac{2\ddot{a_1}}{a_1} + \left(\frac{8\pi G\rho_0 a_0^3}{3c^4 a_1^3}\right)\frac{a_0^3}{a_1^3} = 0$$

Therefore

$$\ddot{a_2} - \frac{\ddot{a_1}a_2}{a_1} - \frac{4\pi G\rho_0}{c^4}\left(\frac{a_0^3}{a_1^3}\right)a_2 = 0 \tag{86}$$

Now since a_1 plays role of a in equation 85 and using equation 78 we get

$$\frac{\ddot{a_1}}{a_1} = \frac{-H_0^2}{2}\left(\frac{a_0^3}{a_1^3}\right)$$

Using the above expression and equation 78 equation 86 can be written as

$$\ddot{a_2} + \frac{H_0^2}{2}\left(\frac{a_0^3}{a_1^3}\right)a_2 - \frac{3H_0^2}{2}\left(\frac{a_0^3}{a_1^3}\right)a_2 = 0$$

$$\Rightarrow \ddot{a_2} - H_0^2\left(\frac{a_0^3}{a_1^3}\right)a_2 = 0$$

Since a_1 plays the role of 'a' in equation 80 we know a_1 as a function of time and using equation 79 to substitute forH_0 we get

$$\ddot{a_2} - \frac{4a_2}{9t^2} = 0 \tag{87}$$

Plotting a graph of a_2 v/s time gives ($a_2^1 \equiv \dot{a_2}$)

Introducing inhomogeneity and anisotropy in $\rho \equiv \rho(t,\theta,\phi)$ as explained above we have

$$\dot{\rho_1} + \dot{\rho_2} = \frac{-3(\dot{a_1} + \dot{a_2})(\rho_1 + \rho_2)}{a_1 + a_2}$$

Note that $\rho_1 \equiv \rho_1(t)$ and $\rho_2 \equiv \rho_2(t,\theta,\phi)$

$$\Rightarrow \dot{\rho_1} + \dot{\rho_2} = \frac{-3(\dot{a_1} + \dot{a_2})(\rho_1 + \rho_2)}{a_1} \left(1 + \frac{a_2}{a_1}\right)^{-1}$$

In the binomial expansion $\left(1 + \frac{a_2}{a_1}\right)^{-1}$ we neglect the terms having the power of a_2 greater than 1

$$\Rightarrow \dot{\rho_1} + \dot{\rho_2} = \frac{-3(\dot{a_1} + \dot{a_2})(\rho_1 + \rho_2)}{a_1} \left(1 - \frac{a_2}{a_1}\right)$$

Approximating $\frac{-3\dot{a_1}\rho_2}{a_1}\left(\frac{-a_2}{a_1}\right)$, $\frac{-3\dot{a_2}\rho_2}{a_1}$, $\frac{-3\dot{a_2}\rho_1}{a_1}\left(\frac{-a_2}{a_1}\right)$ *and* $\frac{-3\dot{a_2}\rho_1}{a_1}\left(\frac{-a_2}{a_1}\right)$ *to* 0 *we get*

$$\dot{\rho_1} + \dot{\rho_2} = -\frac{3\dot{a_1}\rho_1}{a_1} - \frac{3\dot{a_1}\rho_2}{a_1} - \frac{3\dot{a_1}\rho_1}{a_1}\left(\frac{-a_2}{a_1}\right) - \frac{3\dot{a_2}\rho_1}{a_1}$$

Using 81

$$\frac{\dot{\rho_1}}{\rho_1} = \frac{-3\dot{a_1}}{a_1}$$

Therefore

$$\dot{\rho_2} = -\frac{3\dot{a_1}\rho_2}{a_1} + \frac{3\dot{a_1}\rho_1}{a_1}\left(\frac{a_2}{a_1}\right) - \frac{3\dot{a_2}\rho_1}{a_1} \tag{88}$$

We have simulated the process of structure formation in our Universe as a diffusion process running backwards in time.This means that the density of normal matter satisfies the diffusion equation.Actually the process of structure formation of normal matter is anisotropic in space.This means that the ρ_2 term of the density satisfies the diffusion equation i.e,

$$\frac{\partial \rho_2}{\partial t} = D\nabla^2 \rho_2 \tag{89}$$

where D is the diffusion coefficient. But from equation 88 ρ_2 also satisfies

$$\dot{\rho_2} = -\frac{3\dot{a_1}\rho_2}{a_1} + \frac{3\dot{a_1}\rho_1}{a_1}\left(\frac{a_2}{a_1}\right) - \frac{3\dot{a_2}\rho_1}{a_1}$$

Now this is analogous to a diffusion equation since this can be rearranged as

$$\dot{\rho_2} + \frac{3\dot{a_1}\rho_2}{a_1} = \frac{3\dot{a_1}\rho_1}{a_1}\left(\frac{a_2}{a_1}\right) - \frac{3\dot{a_2}\rho_1}{a_1}$$

When analogy is drawn between this equation and the diffusion equation we see that the L.H.S is analogous to diffusion equation and the R.H.S suggests the existence of a source within the system

undergoing diffusion.For the time being let us assume the source term to be absent i.e, remove the perturbation in a.Now in the absence of the perturbation term the equation becomes

$$\dot{\rho_2} = -\frac{3\dot{a_1}\rho_2}{a_1} \tag{90}$$

Compare the above equation with diffusion equation in one dimension for simplicity.Therefore from equation 89 we get

$$\dot{\rho_2} = \frac{\partial \rho_2}{\partial t} = D\frac{\partial^2 \rho_2(z)}{\partial z^2}$$

From the above two equations it follows that

$$D\frac{\partial^2 \rho_2(z)}{\partial z^2} = -\frac{3\dot{a_1}\rho_2}{a_1} \tag{91}$$

This means that D∇^2 acting on ρ_2 is equal $\frac{-3\dot{a_1}}{a_1}$ times ρ_2.Note that $\frac{3\dot{a_1}}{a_1}$ is a constant with respect to position .In such cases the variation of density with position is a sine function and cosine function or a linear combination of both i.e an exponential function.One must also observe that $\frac{3\dot{a_1}}{a_1}$ is just 3 times the value of Hubble's constant at different epochs. In equation 89 if ∇^2 is expanded in spherical polar co-ordinate system as

$$\nabla^2 \equiv \nabla_r^2 + \nabla_\theta^2 + \nabla_\phi^2$$

where ∇_i^2 ; $i = r, \theta , \phi$ are in general functions of all the three variables r , θ , ϕ

If anisotropy and inhomogeneity is taken via $\rho_2(t, \theta, \phi)$ and $a_2(t, \theta, \phi)$ then diffusion equation 89 takes the form

$$\frac{\partial \rho_2}{\partial t} = D(\nabla_r^2 + \nabla_\theta^2 + \nabla_\phi^2)$$

From equation 90 we have

$$\dot{\rho_2} = -\frac{3\dot{a_1}\rho_2}{a_1}$$

From the above two equations we get

$$D(\nabla_r^2 + \nabla_\theta^2 + \nabla_\phi^2) = -\frac{3\dot{a_1}\rho_2}{a_1}$$

Now the solution for the above equation will be of the form

$$\rho_2(t, r, \theta, \phi) = \sum_l c_l J_l(kr) Y_{lm}(\theta, \phi)$$

We consider the sampling of data of CMBR improving in resolution from COBE to WMAP to Planck satellites will give better angular correlations in θ and ϕ. So that a_2 and ρ_2 the anisotropic perturbations of space-time geometry and matter density can be given values at neighboring θ and $\theta \pm d\theta$ and the difference equation converted to a differential equation.Hence a ∇^2 acts on ρ_2 in equation 89.This feature is repeated at scale of super-cluster,cluster and galaxy.

Consider equation 88 and substituting a_1,$\dot{a}_1$ using equation 80 and substituting for rho from equation 82 we arrive at the equation

$$\dot{\rho_2} = \frac{-2\rho_2}{t} + \frac{\rho_0}{a_0}\left(\frac{2a_2}{t} - 3\dot{a_2}\right)\left(\frac{t_0}{t}\right)^{\frac{8}{3}} \tag{92}$$

Considering the equation 90 substituting a_1,$\dot{a}_1$ using equation 80 we get

$$\dot{\rho_2} = \frac{-2\rho_2}{t} \tag{93}$$

This would be the case if in equation 92

$$\frac{2a_2}{t} = 3\dot{a_2} \tag{94}$$

$$\Rightarrow \frac{\dot{a_2}}{a_2} = \frac{2}{3t} \tag{95}$$

From equation 79 one can conclude that

$$H = \frac{2}{3t} = \frac{\dot{a}}{a} = \frac{\dot{a_1} + \dot{a_2}}{a_1 + a_2}$$

From the above two equations it follows that the anisotropic part of Hubble's constant scales in the same way as Hubble when diffusion without a source takes place.This is exactly the situation during WMAP when clustering has just started. From equation 92

$$\dot{\rho_2} = \frac{-2\rho_2}{t} + \frac{\rho_0}{a_0}\left(\frac{2a_2}{t} - 3\dot{a_2}\right)\left(\frac{t_0}{t}\right)^{\frac{8}{3}}$$

If $\dot{a_2}$ is approximated to 0 in the equation 92 which is the present state of Universe (from 1 billion years onwards) where in the change in perturbation is made very small and using equation 80 and equation 82 in the equation 92 we get

$$\dot{\rho_2} + \frac{2\rho_2}{t} = \frac{\rho_1}{a_1}\left(\frac{2a_2}{t}\right)$$

This is an equation in which R.H.S contains the anisotropic part a_2 and roughly models the present state of our Universe.

Considering the equation 91 substituting a_1,$\dot{a}_1$ using equation 80 we get

$$D\frac{\partial^2 \rho_2(z)}{\partial z^2} = \frac{-2\rho_2}{t} \tag{96}$$

By inspection $\rho_{20} exp\left(\frac{-ix}{(\frac{Dt}{2})^{\frac{1}{2}}}\right)$ is a solution where ρ_{20} is a constant

The argument on Differentiation

If we assume that the Universe is filled with dust having then the stress energy tensor $T_{\mu\nu}$ becomes

$$\begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

Then equation 81 gets modified as

$$\frac{\dot{\rho}}{\rho + 3P} = \frac{-3\dot{a}}{a} \tag{97}$$

The equation of state determines P for each form of matter is

$$P = w\rho \tag{98}$$

where w =-1 for dark energy ,$w = \frac{2}{3}$ for dark matter [3],$w = \frac{2}{3}$ for normal matter and $\frac{1}{3}$ for radiation.

Therefore for each kind of matter we have $P_i = w_i\rho_i$ Hence $\rho_i + 3P_i = \rho_i(1 + w_i)$.Hence each kind of matter will distribute differently due to different values of ρ and P.Therefore in equation 97 the value of

$$\rho + 3P = \sum_i (\rho_i(1 + w_i)n_i)$$

where n_i is the number fraction of each type of matter.

Consider equation 88

$$\dot{\rho_2} = -\frac{3\dot{a_1}\rho_2}{a_1} + \frac{3\dot{a_1}\rho_1}{a_1}\left(\frac{a_2}{a_1}\right) - \frac{3\dot{a_2}\rho_1}{a_1}$$

In the above equation ρ_1 is the cumulative of all forms of matter and a_2 is also the cumulative of all forms of matter since it is related to the space time structure and not on the kind of matter.Therefore the only the matter dependent property is ρ_2.Therefore we say that matter does not differentiate itself at the level of ρ_1 But the differentiation occurs at the level of ρ_2 and ρ_2 is a function of position this means that different forms of matter occupy different positions.

ρ_2 is undifferentiated at WMAP stage and $\frac{\rho_2}{\rho}$ is of the order of 10^{-5}. Each component is evolving differently by the time we get to galaxy formation epoch.That means equation 88 is now satisfied independently for each component.Therefore the R.H.S of equation 88 is common but ρ_2 is replaced by $(\rho_{2_i}(1 + w_i)n_i)$. where i runs from 1 to 4.i=1 represents dark energy;i=2 represents dark matter;i=3 represents normal matter;i=4 represents radiation.The analogy of equation 88 with the diffusion equation 89 with the Gaussian solution require that each component has a diffusion coefficient an dσ consistent with the observed data.Hence clustering and differentiation has been explained.

13 Two fluid clustering analog model of the Universe

The Navier Stokes equation of fluid dynamics

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v}\right] = -\nabla P + \eta\nabla^2\vec{v} + \left(\zeta + \frac{\eta}{3}\right) grad(div(\vec{v}))$$

Consider an incompressible fluid i.e, let $div(\vec{v}) = 0$ Therefore the above equation becomes

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v}\right] = -\nabla P + \eta\nabla^2\vec{v}$$

Now at the scale of universe we say that dark energy is a fluid which pushes dark matter and normal matter together to form super- clusters of galaxies but once the structures are formed the

[3]is still to be confirmed through experiments

dark energy it pushes them apart and the super-clusters move away from each other.One must zoom in and find similar process happening at various scales.Assuming that the dark energy is a non-viscous fluid in our model we put $\eta = 0$

$$\rho \left[\frac{\partial \vec{v}}{\partial t} \right] = -\nabla P$$

If $(\vec{v}.\nabla)\vec{v} << \nabla P$ The equation of state for dark energy is $P = -\rho$.Substituting this in the above equation we get

$$\left[\frac{\partial \vec{v}}{\partial t} \right] = \frac{\nabla \rho}{\rho} \tag{99}$$

According to the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \vec{v}) = 0$$

Now $\rho(\nabla v)$

$$\frac{\partial \rho}{\partial t} + \nabla(\rho)\vec{v} = 0$$

But from 99 we substitute $\nabla(\rho)$ in the above equation.Therefore

$$\frac{\partial \rho}{\partial t} = -\left(\rho \frac{\partial \vec{v}}{\partial t} \right).\vec{v}$$

$$\frac{\partial \rho}{\rho} = -\vec{v}.\vec{dv}$$

(We have taken dark energy to be a fluid whose density is invariant with position) Therefore

$$ln(\rho) = \frac{-v^2}{2} + b$$

where ρ

is the density of dark energy,$\vec{v}$ the velocity of dark energy and b is the constant of integration. In case of dark matter the fluid equation is obtained as follows.Consider the Navier-Stokes equation.The clustering of dark and normal matter together suggests that $\nabla P = 0$ and the formation of various kinds of galaxies like spiral suggests that the viscous force is not negligible.But the fluid is assumed to be incompressible i.e, $div(\vec{v}) = 0$.Therefore the equation for dark-matter normal matter mixture becomes

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v} \right] = \eta \nabla^2 \vec{v}$$

The flow of any fluid depends through the boundary conditions,on shape and dimensions of the body moving through the fluid and on its velocity.Since the shape of the body is supposed given,its geometrical properties are determined by one linear dimension,which we denote by l.Let the velocity of the mainstream be u.Then any flow is specified by three parameters, ν,u and l where $nu = \frac{\eta}{\rho}$ is the kinematic viscosity.These quantities have the following dimensions: $\nu = \frac{cm^2}{sec}$, $l = cm$, $u = \frac{cm}{sec}\nu = \frac{cm^2}{sec}$, $l = cm$, $u = \frac{cm}{sec}$. It is easy to verify that only one dimensionless quantity can be

formed from the above three, namely $\frac{ul}{\nu}$.This combination is called the Reynolds number and is denoted by R:

$$R = \frac{\rho ul}{\eta} = \frac{ul}{\nu}$$

In case of the dark-matter,normal matter fluid one can assume the flow to have small Reynolds number because of high value of viscosity of fluid.Now the term $(\vec{v}.\nabla)\vec{v})$ is of the order of magnitude $\frac{u^2}{l}$.The quantity $\left(\frac{\eta}{\rho}\right)\nabla^2\vec{v}$ is of the order of magnitude $\frac{\eta u}{\rho l^2}$. The ratio of the two is just the Reynolds number. Hence the term $((\vec{v}.\nabla)\vec{v})$ may be neglected if the Reynolds number is small,and the equation of motion reduces to

$$\rho\left[\frac{\partial\vec{v}}{\partial t}\right] = \eta\nabla^2\vec{v}$$

One can observe that this is nothing but the diffusion equation for the velocity vector.The velocity vectors which are functions of position and time satisfy the diffusion equation with diffusion constant $\frac{\eta}{\rho}$

-

14 Anisotropy and inhomogeneity of Density in universe

In standard models of Cosmology like Friedmann Robertson Walker (FRW) models the universe is supposed to be homogenous and isotropic.But as we zoom in to smaller scales the inhomogeneity and anisotropy in the form of superclusters, clusters and galaxies appear.

The formation of the *Large Scale Structure* takes place due to perturbation in the density of matter. Our aim is to introduce a small perturbation in the density of the universe. This perturbation is a function of θ, ϕ and t.

The dynamic equations for the *FRW-Model* have the form-

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} \tag{100}$$

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 = -\frac{8\pi G\rho}{3} \tag{101}$$

The above two equation's are not independent of each other and from the above we obtain the relation-

$$\frac{\ddot{R}}{R} = -\left(\frac{\dot{R}}{R}\right)^2 \tag{102}$$

In equation (28) we put

$$\rho \equiv \rho(t) + \delta\rho(\theta, \phi, t)$$

and

$$R \equiv R(t) + \delta R(\theta, \phi, t)$$

where δR and $\delta\rho$ are perturbations of first order in R and ρ.

The equation (28) after solving leads to

$$2\left(\frac{\delta\dot{R}}{\dot{R}} - \frac{\delta R}{R}\right) = \frac{\delta\rho}{\rho} \tag{103}$$

In deriving the above equation the assumption is made that $R(t)$ and $\rho(t)$ follow Eq.. (28). In the above Eq.. $R(t)$, $\dot{R}(t)$ and $\rho(t)$ can be derived from Eq. (28) as functions of time by making the substitution $\rho = \frac{\rho_0 {R_0}^3}{R^3}$. Thus by knowing the variation of $\delta\rho$ with time an ordinary differential equation governing the variation of δR with time is obtained.

In order to obtain the variation of $\delta\rho$ with time we make use of the equation which gives the variation of total density with time.

$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{R}}{R} \tag{104}$$

The above equation is derived by assuming that the total mass of the universe is constant.

Substituting $\frac{\dot{R}}{R}$ from Eq.. (28) in terms of ρ leads to-

$$\dot{\rho} = -3\rho^{3/2}\sqrt{\frac{8\pi G}{3}} \tag{105}$$

In which after substituting ρ=$\rho(t) + \delta\rho(\theta, \phi)$ and assuming that $\rho(t)$ follows Eq.. (31) we get

$$\frac{\delta\dot{\rho}}{\delta\rho} = -\frac{9}{2}\sqrt{\frac{8\pi G\rho}{3}} \tag{106}$$

From Eq.. (33) the time variation of $\delta\rho$ can be obtained and thus the time variation of δR can be obtained.

15 Distribution and differentiation of various forms of matter in the Universe

Consider the differential equation of a_2v/st

$$\ddot{a_2} - \frac{4a_2}{9t^2} = 0$$

The solution of the above equation is $\frac{c_1}{t^{\frac{1}{3}}} + c_2 t^{\frac{4}{3}}$ where c_1 and c_2 are integration constants. One can see that this function attains an extremum at $\frac{4c_2}{c_1}$

Supposing the initial conditions were (i.e $a_2 = 0 \; at \; t = 0$ we get the solution as

$$a_2 = c(\theta, \phi) t^{\frac{4}{3}} \tag{107}$$

Consider the differential equation ρ_2v/s t

$$\dot{\rho_2} = -\frac{3\dot{a_1}\rho_2}{a_1} + \frac{3\dot{a_1}\rho_1}{a_1}\left(\frac{a_2}{a_1}\right) - \frac{3\dot{a_2}\rho_1}{a_1}$$

Substituting equation 107 in the above equation along with

$$\rho = \frac{\rho_0 t_0^2}{t^2}$$

and

$$a_1 = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

Therefore

$$\dot{\rho_2} = -\frac{3\frac{d}{dt}\left(a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right)\rho_2}{a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}} + \frac{3\frac{d}{dt}\left(a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right)\frac{\rho_0 t_0^2}{t^2}}{a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}}\left(\frac{c(\theta,\phi)t^{\frac{4}{3}}}{a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}}\right) - \frac{3\frac{d}{dt}\left(c(\theta,\phi)t^{\frac{4}{3}}\right)\frac{\rho_0 t_0^2}{t^2}}{a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}}$$

$$\dot{\rho_2} = -\frac{2\rho_2}{t} + \frac{2c(\theta,\phi)\rho_0 t_0^{\frac{8}{3}}}{a_0 t^{\frac{7}{3}}} - \frac{4c(\theta,\phi)\rho_0 t_0^{\frac{8}{3}}}{a_0 t^{\frac{7}{3}}}$$

We have $t_0 = 13.7 \times 10^9 \; years \; , \; \rho_0 = 2.11 \times 10^{-29} \; kg/m^3 \quad and \quad a_0 = 1$ Hence

$$\dot{\rho_2} = \frac{2\rho_2}{t} + \frac{4.51 \times 10^{18} c(\theta,\phi)}{t^{\frac{7}{3}}}$$

$$\rho_2 = exp\left(-\int \frac{2}{t}dt\right)\left[\int\left(\frac{4.51\times 10^{18}c(\theta,\phi)}{t^{\frac{7}{3}}} exp\left(\int \frac{2}{t}dt\right)\right)dt + K\right]$$

$$\rho_2 = t^{-2}\left[4.51 \times 10^{18} c(\theta,\phi) \; \int\left(t^{2-\frac{7}{3}}\right)dt + K\right]$$

$$\rho_2 = \left[\frac{4.51 \times 10^{18} c(\theta,\phi) \; t^{\frac{-4}{3}}}{2 - \frac{4}{3}} + Kt^{-2}\right]$$

$$\rho_2 = \left[6.765 \times 10^{18} c(\theta,\phi) \; t^{\frac{-4}{3}} + Kt^{-2}\right]$$

If we substitute the initial conditions at t=WMAP i.e $\frac{a_2}{a_1 + a_2} = 10^{-5}$ we get

$$\frac{\frac{c_1}{t^{\frac{1}{3}}} + c_2 t^{\frac{4}{3}}}{\frac{c_1}{t^{\frac{1}{3}}} + c_2 t^{\frac{4}{3}} + a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}} = 10^{-5}$$

$$2.00276 \times 10^{22} c_2 + 4.72702 c_1 = 7.82709 \times 10^8 \tag{108}$$

However if we substitute a_2 to be $\frac{c_1}{t^{\frac{1}{3}}} + c_2 t^{\frac{4}{3}}$ in the differential equation of ρ_2 (if one is not sure about the initial conditions of a_2) we get

$$\dot{\rho_2} = -\frac{3\frac{d}{dt}\left(a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right)\rho_2}{a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}} + \frac{3\frac{d}{dt}\left(a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right)\frac{\rho_0 t_0^2}{t^2}}{a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}}\left(\frac{\frac{c_1}{t^{\frac{1}{3}}} + c_2 t^{\frac{4}{3}}}{a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}}\right) - \frac{3\frac{d}{dt}\left(\frac{c_1}{t^{\frac{1}{3}}} + c_2 t^{\frac{4}{3}}\right)\frac{\rho_0 t_0^2}{t^2}}{a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}}$$

$$\dot{\rho_2} = -\frac{2\rho_2}{t} + \frac{2c_1\rho_0 t_0^{\frac{8}{3}}}{a_0 t^4} + \frac{c_1\rho_0 t_0^{\frac{8}{3}}}{a_0 t^4} + \frac{2c_2\rho_0 t_0^{\frac{8}{3}}}{a_0 t^{\frac{7}{3}}} - \frac{4c_2\rho_0 t_0^{\frac{8}{3}}}{a_0 t^{\frac{7}{3}}}$$

We have $t_0 = 13.7 \times 10^9\ years$, $\rho_0 = 2.11 \times 10^{-29}\ kg/m^3 \quad and \quad a_0 = 1$ Hence

$$\dot{\rho_2} = \frac{-2\rho_2}{t} + \frac{6.76502 \times 10^{18} c_1}{t^4} + \frac{(-4.51001 \times 10^{18} c_2)}{t^{\frac{7}{3}}}$$

$$\dot{\rho_2} + \frac{2\rho_2}{t} = \frac{6.76502 \times 10^{18} c_1}{t^4} + \frac{(-4.51001 \times 10^{18} c_2)}{t^{\frac{7}{3}}}$$

Call 2 as k_1,$6.76502 \times 10^{18} c_1$ as k_2 and$-4.51001 \times 10^{18} c_2$ as k_3. Therefore the above equation becomes

$$\dot{\rho_2} + \frac{k_1}{t} = \frac{k_2}{t^4} + \frac{k_3}{t^{\frac{7}{3}}}$$

The above equation is a linear first order differential equation whose solution is

$$\rho_2 = \left(\frac{k_2 t^{-3}}{k_1 - 3}\right) + \left(\frac{k_3 t^{\frac{-4}{3}}}{k_1 - \frac{4}{3}}\right) + k t^{-k_1}$$

where k is the constant of integration.Substituting the values of k_1, k_2 and k_3 respectively we get

$$\rho_2 = \left(-6.76502 \times 10^{18} c_1 t^{-3}\right) + \left(-6.765 \times 10^{18} c_2 t^{\frac{-4}{3}}\right) + k t^{-2}$$

where k is the constant of integration.
We know that at $t = WMAP = 300,000$ years

$$\frac{\rho_2}{\rho_1 + \rho_2} = 10^{-5} \tag{109}$$

ρ_2 at $t = 300,000$ years is calculated to be $-7.9724 \times 10^{-21} c_1 - 33.778 c_2 + 1.116 \times 10^{-26}$ and ρ_1 at $t = 300,000$ years is equal to 4.40029×10^{-20}. Therefore at t=WMAP substituting ρ_1 and ρ_2 in 109 we get

$$3.3778 \times 10^6 c_2 - 7.9724 \times 10^{-16} c_1 + 1.116 \times 10^{-26} = 4.40029 \times 10^{-20}$$

Now c_1 and c_2 are actually functions of θ and ϕ.Therefore imagine $c_1 = \alpha f(\theta, \phi)$ and $c_2 = \beta g(\theta, \phi))$

16 Calculation of Einstein tensor for the modified Friedmann Robertson Walker metric

Let us call a[t] as R[t]. The modified Robertson Walker metric is given by

$$(ds)^2 = (dt)^2 - [R(t)+b(t,\theta,\phi)]^2(dr)^2 - [R(t)+b(t,\theta,\phi)]^2(r)^2(d\theta)^2 - [R(t)+b(t,\theta,\phi)]^2(r)^2 sin^2(\theta)(d\phi)^2$$

where $b(t,\theta,\phi)$ is the perturbation introduced in $R(t)$ Then the components of the Einstein Tensor for the modified Robertson Walker metric which introduces inhomogeneity is

Below R is a function of t and b is a function of t , θ , ϕ.Note that for each of the terms below the term in the first bracket is nothing but the terms of the Einstein tensor for the Robertson-Walker metric.Also in evaluating the Einstein tensor a binomial expansion was made and terms involving higher powers of b or the derivatives of b have been neglected. The complete Einstein tensor was evaluated using Mathematica.

17 Einstein tensor for perturbed FRW metric

$$G_{00} = \frac{-1}{r^2R^4}\left[\begin{array}{c} \left[[3(rRR^{'})^2\right] + b\left(6R(rR^{'})^2 - 12(rR^{'})^2R\right) + \frac{\partial b}{\partial t}(6(rR)^2R^{'}) \\ +\frac{\partial b}{\partial \theta}(-2Rcot(\theta)) + \frac{\partial^2 b}{\partial \theta^2}(-2R) + \frac{\partial^2 b}{\partial \phi^2}(-2Rcosec^2(\theta)) \end{array}\right]$$

$$G_{01} = 0$$

$$G_{02} = \frac{2}{R^2}\left[-R^{'}\left(\frac{\partial b}{\partial \theta}\right) + R\left(\frac{\partial^2 b}{\partial t \partial \theta}\right)\right]$$

$$G_{03} = \frac{2}{R^2}\left[-R^{'}\left(\frac{\partial b}{\partial \phi}\right) + R\left(\frac{\partial^2 b}{\partial t \partial \phi}\right)\right]$$

$$G_{10} = 0$$

$$G_{11} = \frac{1}{(rR)^2}\left[\begin{array}{c} \left[(rRR^{'})^2 + 2r^2R^3R^{''}\right] + b\left(2R(rR^{'})^2 + 6(rR)^2R^{''} + \frac{-2}{R}\left((rRR^{'})^2 + 2r^2R^3R^{''}\right)\right) \\ +\frac{\partial b}{\partial t}(2(rR)^2R^{'}) + \frac{\partial b}{\partial \theta}(-Rcot(\theta)) + \frac{\partial^2 b}{\partial t^2}(2r^2R^3) + \frac{\partial^2 b}{\partial \theta^2}(-R) + \frac{\partial^2 b}{\partial \phi^2}(-Rcosec^2\theta) \end{array}\right]$$

$$G_{12} = \frac{-1}{rR}\left(\frac{\partial b}{\partial \theta}\right)$$

$$G_{13} = \frac{-1}{rR}\left(\frac{\partial b}{\partial \phi}\right)$$

$$G_{20} = \frac{2}{R^2}\left[-R^{'}\left(\frac{\partial b}{\partial \theta}\right) + R\left(\frac{\partial^2 b}{\partial t \partial \theta}\right)\right]$$

$$G_{21} = \frac{-1}{rR}\left(\frac{\partial b}{\partial \theta}\right)$$

$$G_{22} = \frac{1}{(rR)^2}\left[\begin{array}{c} \left[(rRR^{'})^2 + 2r^2R^3R^{''}\right] + b\left(2R(rR^{'})^2 + 6(rR)^2R^{''} + \frac{-2}{R}\left((rRR^{'})^2 + 2r^2R^3R^{''}\right)\right) \\ +\frac{\partial b}{\partial t}(2(rR)^2R^{'}) + \frac{\partial b}{\partial \theta}(-Rcot(\theta)) + \frac{\partial^2 b}{\partial t^2}(2r^2R^3) + \frac{\partial^2 b}{\partial \phi^2}(-Rcosec^2\theta) \end{array}\right]$$

$$G_{23} = \frac{1}{R}\left[\left(\frac{\partial b}{\partial \phi}\right)(-cot\theta) + \frac{\partial^2 b}{\partial\theta\partial\phi}\right]$$

$$G_{30} = \frac{2}{R^2}\left[-R^{'}\left(\frac{\partial b}{\partial \phi}\right) + R\left(\frac{\partial^2 b}{\partial t\partial\phi}\right)\right]$$

$$G_{31} = \frac{-1}{rR}\left(\frac{\partial b}{\partial \phi}\right)$$

$$G_{32} = \frac{1}{R}\left[\left(\frac{\partial b}{\partial \phi}\right)(-cot\theta) + \frac{\partial^2 b}{\partial\theta\partial\phi}\right]$$

$$G_{33} = \frac{1}{R^2}\left[\begin{array}{c} \left[\left(rRR^{'}sin\theta\right)^2 + 2r^2R^3sin^2\theta R^{''}\right] \\ +b\left(2R(rR^{'}sin\theta)^2 + 6R''(rRsin\theta)^2 - \frac{2}{R}\left(rRR^{'}sin\theta\right)^2 + 2r^2R^3sin^2\theta R^{''}\right) \\ +\frac{\partial b}{\partial t}(2R(rR^{'}sin\theta)^2) + \frac{\partial^2 b}{\partial t^2}(2r^2R^3sin^2\theta) \end{array}\right]$$

If we consider the standard stress-energy tensor (which is dust with pressure) in which the off diagonal terms are zero then $b(t, \theta, \phi) = 0$ which removes the inhomogeneity which is wanted in the first place.So one must think of an appropriate stress energy tensor which when equated to the Einstein tensor gives a solution for $b(t, \theta, \phi)$

The off diagonal terms in Einstein's equation are set to zero in this approximation as case (1), and in the right hand side, the off diagonal elements of the stress energy tensor are set to zero. Consider only the diagonal terms.

18 Approximate solution

$$R = a_1 = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

Now since $a_0 = 1$

$$\Longrightarrow R^{'} = \frac{2}{3t_0}\left(\frac{t}{t_0}\right)^{\frac{-1}{3}}$$

and

$$\Longrightarrow R^{''} = \frac{-2}{9t_0^2}\left(\frac{t}{t_0}\right)^{\frac{-4}{3}}$$

Also

$$b = a_2 = c(\theta, \phi)t^{\frac{4}{3}}$$
$$\frac{\partial b}{\partial t} = \frac{4}{3}c(\theta, \phi)t^{\frac{1}{3}}$$
$$\frac{\partial^2 b}{\partial t^2} = \frac{4}{9}c(\theta, \phi)t^{\frac{-2}{3}}$$

Also

$$\frac{\partial b}{\partial \theta} = \frac{\partial c(\theta, \phi)}{\partial \theta}t^{\frac{4}{3}}$$

Neglecting the higher derivatives of θ and ϕ in G_{00} we get

$$G_{00} = \frac{-1}{r^2R^4}\left[\begin{array}{c}\left(3(rRR^{'})^2\right) + b\left(-6(rR^{'})^2R\right) + \frac{\partial b}{\partial t}(6(rR)^2R^{'}) \\ +\frac{\partial b}{\partial \theta}(-2Rcot(\theta))\end{array}\right]$$

$$G_{00} = \frac{-1}{r^2\left[\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right]^4}\left[\begin{array}{c}\left(3\left(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{2}{3t_0}\left(\frac{t}{t_0}\right)^{\frac{-1}{3}}\right)^2\right) + c(\theta,\phi)t^{\frac{4}{3}}\left(-6\left(\frac{2r}{3t_0}\left(\frac{t}{t_0}\right)^{\frac{-1}{3}}\right)^2\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right) \\ +\frac{4c(\theta,\phi)t^{\frac{1}{3}}}{3}\left(6(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}})^2\frac{2}{3t_0}\left(\frac{t}{t_0}\right)^{\frac{-1}{3}}\right) + \frac{\partial c(\theta,\phi)}{\partial\theta}t^{\frac{4}{3}}\left(-2\left(\frac{t}{t_0}\right)^{\frac{2}{3}}cot(\theta)\right)\end{array}\right]$$

Therefore G_{00} simplifies to give

$$\frac{-1}{r^2}\left(\frac{t}{t_0}\right)^{\frac{8}{3}}\left[\frac{4r^2}{3t_0^2}\left(\frac{t}{t_0}\right)^{\frac{2}{3}} + \frac{8cr^2t^{\frac{4}{3}}}{3t_0^2} - 2cot(\theta)t^{\frac{4}{3}}\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{\partial c(\theta,\phi)}{\partial\theta}\right]$$

Neglecting the higher derivatives of θ and ϕ in G_{11} we get

$$G_{11} = \frac{1}{(rR)^2}\left[\begin{array}{c}\left[(rRR^{'})^2 + 2r^2R^3R^{''}\right] + b\left(2R(rR^{'})^2 + 6(rR)^2R^{''} + \frac{-2}{R}\left((rRR^{'})^2 + 2r^2R^3R^{''}\right)\right) \\ +\frac{\partial b}{\partial t}(2(rR)^2R^{'}) + \frac{\partial b}{\partial\theta}(-Rcot(\theta)) + \frac{\partial^2 b}{\partial t^2}(2r^2R^3)\end{array}\right]$$

$$G_{11} = \frac{1}{\left(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right)^2}\left[\begin{array}{c}\left[\left(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{2}{3t_0}\left(\frac{t}{t_0}\right)^{\frac{-1}{3}}\right)^2 + 2r^2\left[\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right]^3\frac{-2}{9t_0^2}\left(\frac{t}{t_0}\right)^{\frac{-4}{3}}\right] \\ +c(\theta,\phi)t^{\frac{4}{3}}\left(2\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\left(\frac{2r}{3t_0}\left(\frac{t}{t_0}\right)\right)^2 + 6\left(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right)^2\frac{-2}{9t_0^2}\left(\frac{t}{t_0}\right)^{\frac{-4}{3}}\right) \\ +c(\theta,\phi)t^{\frac{4}{3}}\left(\frac{-2}{\left(\frac{t}{t_0}\right)^{\frac{2}{3}}}\left(\left(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{2}{3t_0}\left(\frac{t}{t_0}\right)^{\frac{-1}{3}}\right)^2 + 2r^2\left[\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right]^3\frac{-2}{9t_0^2}\left(\frac{t}{t_0}\right)^{\frac{-4}{3}}\right)\right) \\ +\frac{4}{3}c(\theta,\phi)t^{\frac{1}{3}}\left(2(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}})^2\frac{2}{3t_0}\left(\frac{t}{t_0}\right)^{\frac{-1}{3}}\right) \\ -\frac{\partial c(\theta,\phi)}{\partial\theta}t^{\frac{4}{3}}\left(\left(\frac{t}{t_0}\right)^{\frac{2}{3}}cot(\theta)\right) \\ +\frac{4}{9}c(\theta,\phi)t^{\frac{-2}{3}}\left(2r^2\left[\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right]^3\right)\end{array}\right]$$

$$G_{11} = \frac{1}{\left(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right)^2}\left[\frac{28cr^2t^{\frac{4}{3}}}{9t_0^2} - \frac{\partial c(\theta,\phi)}{\partial\theta}t^{\frac{4}{3}}\left(\left(\frac{t}{t_0}\right)^{\frac{2}{3}}cot(\theta)\right) + \frac{8cr^2t^{\frac{-2}{3}}}{9}\left(\frac{t}{t_0}\right)^2\right]$$

Observation yields

$$G_{11} + \frac{R}{\left(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right)^2}\left[\frac{\partial^2 b}{\partial\theta^2}\right] = G_{22}$$

Neglect the higher derivatives of $b = c(\theta,\phi)t^{\frac{4}{3}}$ yields

$$G_{11} = G_{22}$$

G_{33}does not yield a differential equation for $b(\theta,\phi)$ and hence it is redundant for this approximation.

Therefore

$$G_{00} = \frac{-1}{r^2}\left(\frac{t}{t_0}\right)^{\frac{-8}{3}}\left[\frac{4r^2}{3t_0^2}\left(\frac{t}{t_0}\right)^{\frac{2}{3}} + \frac{8cr^2t^{\frac{4}{3}}}{3t_0^2} - 2cot(\theta)t^{\frac{4}{3}}\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{\partial c(\theta,\phi)}{\partial\theta}\right]$$

$$G_{11} = G_{22} = \frac{1}{\left(r\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\right)^2}\left[\frac{28ct^{\frac{4}{3}}r^2}{9t_0^2} - \frac{\partial c(\theta,\phi)}{\partial\theta}t^{\frac{4}{3}}\left(\left(\frac{t}{t_0}\right)^{\frac{2}{3}}cot(\theta)\right) + \frac{8cr^2t^{\frac{-2}{3}}}{9}\left(\frac{t}{t_0}\right)^2\right]$$

Substitute for t the WMAP time which is $3\times10^5\times365\times24\times60\times60$ and for t_0 $13.6\times10^9\times365\times24\times60\times60$ seconds. These in the units $c=1$ will be measured in meters i.e

$$t_{WMAP} = 3\times10^5\times365\times24\times60\times60\times3\times10^8\ meters = 2.84\times10^{21}\ meters$$

$$t_0 = 13.6\times10^9\times365\times24\times60\times60\times3\times10^8\ meters = 1.29\times10^{26}\ meters$$

The first term in G_{00} is far less in magnitude than the other terms. Hence

$$G_{00} = \frac{-1}{r^2}\left(\frac{t}{t_0}\right)^{\frac{-8}{3}}\left[\frac{8cr^2t^{\frac{4}{3}}}{3t_0^2} - 2cot(\theta)t^{\frac{4}{3}}\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{\partial c(\theta,\phi)}{\partial\theta}\right]$$

$$G_{11} = G_{22} = \frac{-1}{r^2}\left(\frac{t}{t_0}\right)^{\frac{-8}{3}}\left[\frac{4cr^2t^{\frac{4}{3}}}{t_0^2} - cot(\theta)t^{\frac{4}{3}}\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{\partial c(\theta,\phi)}{\partial\theta}\right]$$

Now in $c=1$ units $G_{00} = 8\pi G(\rho_1+\rho_2)$. This includes the first and second order terms. The G_{00} which we obtained is second order, and approximating $\rho_2\approx0$.

$8\pi G\rho_2$ has a very small value compared to terms on the L.H.S.

19 case 1

$$\frac{-1}{r^2}\left(\frac{t}{t_0}\right)^{\frac{-8}{3}}\left[\frac{8cr^2t^{\frac{4}{3}}}{3t_0^2} - 2cot(\theta)t^{\frac{4}{3}}\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{\partial c(\theta,\phi)}{\partial\theta}\right] = 0$$

$$\implies \left[\frac{8cr^2t^{\frac{4}{3}}}{3t_0^2} - 2cot(\theta)t^{\frac{4}{3}}\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{\partial c(\theta,\phi)}{\partial\theta}\right] = 0$$

$$\implies \frac{4r^2\left(t_{WMAP}\right)^{\frac{-2}{3}}\left(t_0\right)^{\frac{-4}{3}}}{3}\left[tan(\theta)\,\partial\theta\right] = \frac{\partial\left[c(\theta,\phi)\right]}{c(\theta,\phi)}$$

$$\implies \frac{4r^2\left(t_{WMAP}\right)^{\frac{-2}{3}}\left(t_0\right)^{\frac{-4}{3}}}{3}\int tan(\theta)\,\partial\theta = \int\frac{\partial\left[c(\theta,\phi)\right]}{c(\theta,\phi)}$$

$$\implies 1.024\times10^{-49}r^2ln(sec(\theta)) + ln(g(\phi)) = ln(c(\theta,\phi))$$

where $g(\phi)$ is the constant of integration.

$$\Longrightarrow c(\theta,\phi) = g(\phi)[sec\theta]^{1.024\times 10^{-49} r^2}$$

Now in $c = 1$ units $G_{11} = G_{22} = 8\pi G(P_1 + P_2)$. Now the G_{11} and G_{22} which we obtained is called second order and approximating i.e $P_2 \approx 0$ then $8\pi G P_2$ has a very small value compared to terms on the L.H.S. Hence we get

$$\frac{-1}{r^2}\left(\frac{t}{t_0}\right)^{\frac{-8}{3}}\left[\frac{4cr^2 t^{\frac{4}{3}}}{t_0^2} - cot(\theta) t^{\frac{4}{3}}\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{\partial c(\theta,\phi)}{\partial\theta}\right] = 0$$

$$\Longrightarrow 4r^2\,(t_{WMAP})^{\frac{-2}{3}}\,(t_0)^{\frac{-4}{3}}\,[tan(\theta)\,\partial\theta] = \frac{\partial\,[c(\theta,\phi)]}{c(\theta,\phi)} \tag{110}$$

$$\Longrightarrow 4r^2\,(t_{WMAP})^{\frac{-2}{3}}\,(t_0)^{\frac{-4}{3}}\int tan(\theta)\,\partial\theta = \int\frac{\partial\,[c(\theta,\phi)]}{c(\theta,\phi)}$$

$$\Longrightarrow 3\times 1.024\times 10^{-49} r^2 ln(sec(\theta)) + ln(h(\phi)) = ln(c(\theta,\phi))$$

where $h(\phi)$ is the constant of integration.

$$\Longrightarrow c(\theta,\phi) = h(\phi)[sec\theta]^{3\times 1.024\times 10^{-49} r^2}$$

Now for $r = r_{WMAP} = r_0\left(\frac{t_{WMAP}}{t_0}\right)^{\frac{2}{3}} = 7.139\times 10^{22}$. Finally

$$\Longrightarrow c(\theta,\phi) = h(\phi)[sec\theta]^{4.68\times 10^{-3}}$$

Using the G_{11} equation one gets

$$c(\theta,\phi) = g(\phi)[sec\theta]^{1.56\times 10^{-3}}$$

Therefore

$$c(\theta,\phi) = g(\phi)[sec\theta]^{1.56\times 10^{-3}} = h(\phi)[sec\theta]^{4.68\times 10^{-3}}$$

Now in obtaining $c(\theta,\phi)$ we have taken $\frac{\partial^2 b}{\partial\theta^2} \approx 0$ and neglected this term. So this expression is valid for those values of θ where $\frac{\partial^2 c(\theta,\phi)}{\partial\theta^2} \approx 0$ i.e $\frac{\partial c(\theta,\phi)}{\partial\theta} \approx$ constant and any deviation will be linear in θ. Inserting the solution we find that the first derivative is proportional approximately to $tan(\theta)$ equals θ for small θ. This happens for those values of θ for which $c(\theta,\phi)tan(\theta) \approx$ constant, which is true for values of θ close to 0. Hence the given function describes anisotropy the best for values of θ around 0.

To evaluate the functions of ϕ one must need another equation in $c(\theta,\phi)$ v/s ϕ for which one must take higher order perturbations. But as of now $b(t,\theta,\phi) = g(\phi)[sec\theta]^{1.56\times 10^{-3}} t^{\frac{-4}{3}} = h(\phi)[sec\theta]^{4.68\times 10^{-3}} t^{\frac{-4}{3}}$. The off diagonal terms of G_{ij} were set to zero. But since we now have the expression for $b(t,\theta,\phi)$ we can now substitute it in the off diagonal terms say G_{12} and G_{21} and since $G_{12} = 8\pi G T_{12}$ and $G_{21} = 8\pi G T_{21}$ we have now actually got two terms for our stress energy tensor. i.e

$$T_{12} = T_{21} = \frac{-g(\phi)(\alpha sec^{\alpha}\theta tan\theta) t_{wmap}^{\frac{4}{3}}}{r_{wmap}}\left[\frac{t_{wmap}}{t_0}\right]^{-\frac{2}{3}}$$

where $\alpha = 1.56 \times 10^{-3}$. Similarly the other components of the stress energy tensor can be determined. They would have derivatives of $h(\phi)$. T_{12} and T_{21} involve $h(\phi)$. But it can be determined by not neglecting $\frac{\partial^2 b}{\partial \phi^2}$. Thus the stress energy tensor can be completely determined and seen to correspond to inhomogeneity. This can be interpreted as introducing perturbations in the ideal fluid contained in Universe to obtain the non-ideal case.

20 case 2

In the above calculation we have ρ_1 non zero for first order and $\rho_2 = 0$. But $\rho_2 = G_{00}$ for second order. Here

$$\rho = \frac{\rho_0 t_0^2}{t^2}$$

$$\left[6.765 \times 10^{18} \times (3 \times 10^8)^{\frac{8}{3}} c(\theta, \phi)\ t^{\frac{-4}{3}} + Kt^{-2}\right]$$

where $\rho_0 = 2.11 \times 10^{-29}$ and K is a constant $t = t_{wmap}$ and $t_0 = presenttime$

$$G_{00} = \frac{-1}{r^2}\left(\frac{t}{t_0}\right)^{\frac{8}{3}}\left[\frac{4r^2}{3t_0^2}\left(\frac{t}{t_0}\right)^{\frac{2}{3}} + \frac{8cr^2t^{\frac{4}{3}}}{3t_0^2} - 2cot(\theta)t^{\frac{4}{3}}\left(\frac{t}{t_0}\right)^{\frac{2}{3}}\frac{\partial c(\theta,\phi)}{\partial \theta}\right]$$

$$= \rho_1 + \rho_2 = \frac{\rho_0 t_0^2}{t^2} = \left[6.765 \times 10^{18} \times (3 \times 10^8)^{\frac{8}{3}} c(\theta, \phi)\ t^{\frac{-4}{3}} + Kt^{-2}\right]$$

which yields

$$(cot(\theta) \times 10^8)\frac{\partial c(\theta,\phi)}{\partial \theta} - c(\theta,\phi)(69 \times 10^{12}) + Kt^{-2} + 4.35 \times 10^{-20} = 0$$

This equation can again be solved for $c(\theta, \phi)$, and hence for b and the complete Einstein tensor and stress energy tensor can be determined. The model is solved at WMAP time.

21 Conclusion

The correlation between the 2nd order gravity wave analysis which gives the space time 'wrinkles' on the FRW background and the density and curvature perturbations of second order that give the matter distribution on FRW background is expected, as the Einstein equations remain valid order by order. The correlation of the observed WMAP radiation data to the theoretical model is also suggested. The WMAP data has been explained by other models such as inflation driven inhomogeneity. In this work the contribution of gravity waves , non linearly coupled on the FRW background, organizing the density and curvature inhomogeneity was investigated.

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